PROBLEMS ON REPRESENTATION THEORY

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1. Group rings and representations

Be comfortable with the slogan: "representations of G are equivalent to modules over $k[G]$".

(1) 2010 Fall M2
(2) 2010 Fall A2

2. Linear algebra

(1) 2011 Spring A5
(2) 2013 Fall M1

3. Representations of Finite groups

Here's a quick "cheat sheet" of useful facts about the character table of a finite group:

- The number of irreducible characters of G coincides with the number of conjugacy classes of G. In fact, the irreducible characters form a basis for the vector space of class functions.
- Note that $\chi_V(1) = \dim V$. Then the sum of the dimensions squared over all irreducible V is $|G|$:
$$|G| = \sum_{\chi} |\chi(1)|^2.$$
- Characters of nonisomorphic irreducible representations are orthogonal:
$$\sum_{g \in |G|} \chi(g)\overline{\chi'(g)} = 0 \text{ if } \chi \not\simeq \chi'.$$
- The regular representation has character supported on the identity element, and breaks up as $\bigoplus_V V^{\dim V}$ where the sum is over all irreducibles. Said differently,
$$\sum_{\chi} \chi(1)\chi(g) = \begin{cases} |G| & g = e \\ 0 & \text{otherwise} \end{cases}$$

(1) 2011 Fall A2
(2) 2013 Spring A3
(3) 2011 Fall M4

4. Modular representations

These problems are about representations over fields of characteristic p. In particular, the theory of characters (as in the previous section) does not apply.

(1) 2010 Spring A3
(2) 2011 Fall A3
(3) 2012 Spring M4

5. Induction and Restriction

For this you should know:

- The definition of induced representations. If $H \subset G$ is a subgroup and V is a representation of H over a field k, then $k[G] \otimes_{k[H]} V$ is the induced representation of G.
- Frobenius reciprocity: $\mathrm{Hom}_G(k[G] \otimes_{k[H]} V, W) = \mathrm{Hom}_H(V, W)$.

(1) 2016 Spring M3
(2) 2016 Fall M4