MATH 249 PROBLEM SET 3 (DUE OCTOBER 17)

(1) Show that if P is a CW poset, then P is graded, i.e. for all $x \in P$, all maximal chains in $[\hat{0}, x]$ have the same length.

(2) Show that the length of an element w in the symmetric group S_n equals the number of inversions, i.e. it is the cardinality of the set $\{(i,j) \mid 1 \leq i < j \leq n \text{ and } w(i) > w(j)\}$.

(3) Show that the Bruhat order on the symmetric group is *thin*, i.e. every rank 2 interval is a diamond (equivalently every rank 2 interval has precisely 4 elements).

(4) Let G be a finite graph. Let E be the set of all edges in G and let $\mathcal{I}$ be the collection of subsets of E that do not contain all the edges of a cycle of G. Use a graph-theoretic argument to show that $(E, \mathcal{I})$ is a matroid (with $\mathcal{I}$ representing the independent sets).