

1 Introduction.

Roughly speaking a planar algebra should be any set together with operations *naturally* indexed by planar diagrams. In the late 1990's the author introduced in [1] a precise but very special notion of planar algebra designed for the study of subfactors of finite index. This narrow use of the term planar algebra was an attempt to avoid a nightmare of qualifying adjectives such as "unshaded oriented finite dimensional speherical C^* planar algebra". In hindsight this was a mistake as other cases are proving interesting beyond subfactors (e.g. [2]). With this complete revision of [1], together with the addition of much new material, we are attempting to redress the situation and present a versatile and generally useful notion of planar algebra.

Besides subfactors there were many structures that motivated planar algebras. Let us mention a few hoping that the reader will be familiar with at least one of them. Conway's tangles and linear skein theory, Kauffman's diagrammatics for the Temperley-Lieb algebra, two dimensional statistical mechanical models, Penrose's diagrammatic tensor calculus, Van Kampen diagrams, and TQFT were all contributing ideas. And the planar algebra idea was certainly in the air. Kuperberg's (earlier) notion of spider is was very close in spirit to our very first attempt to define planar algebras. Tensor categories and 2-categories also intersect hugely with planar algebras.