Here is the correct version of

$$\sum_{n=1}^{\infty} n^2 x^n$$

Start with $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ and differentiate once to get

i) $\dfrac{1}{(1-x)^2} = \displaystyle\sum_{n=1}^{\infty} n x^{n-1}$

Differentiate again to get

$$\frac{2}{(1-x)^3} = \sum_{n=2}^{\infty} n(n-1)x^{n-2}.$$

Observing that $n(n-1) = n^2 - n$ and rearranging we get

$$\sum_{n=2}^{\infty} n^2 x^{n-2} = \frac{2}{(1-x)^3} + \sum_{n=2}^{\infty} n x^{n-2}$$

Multiplying by x^2 gives almost what we want on the left hand side:

$$\sum_{n=2}^{\infty} n^2 x^n = \frac{2x^2}{(1-x)^3} + x^2 \sum_{n=2}^{\infty} n x^{n-2} = \frac{2x^2}{(1-x)^3} + x \sum_{n=2}^{\infty} n x^{n-1}$$

Notice that the sum on the left hand side is missing the "n=1" term in order to be what we are looking for and the sum on the right is missing the "n=1" term in order to he the same as the sum in equation i). In both cases the "n=1" term is just x so we may add it to the left and right hand sides to get:

$$\sum_{n=1}^{\infty} n^2 x^n = x + \sum_{n=2}^{\infty} n^2 x^n == \frac{2x^2}{(1-x)^3} + x + x \sum_{n=2}^{\infty} n x^{n-1} = \frac{2x^2}{(1-x)^3} + x \sum_{n=1}^{\infty} n x^{n-1}$$

Now using equation i) we get

$$\sum_{n=1}^{\infty} n^2 x^n = \frac{2x^2}{(1-x)^3} + \frac{x}{(1-x)^2} = \frac{x(1+x)}{(1-x)^3}$$

If you don't believe it try adding 10 or so terms in the sum with your calculator for $x = 1/2$ and see how close it is to the right hand side!