Here are the details of the example on surface of revolution we did not finish in class:

Find the area of the surface obtained by rotating the curve

$$x = 1 + 2y^2$$

for $1 \le y \le 2$ about the x axis.

First way: as a "y" integral:

$$2\pi \int_1^2 y\sqrt{1 + (\frac{dx}{dy})^2}dy$$

$$= 2\pi \int_1^2 y\sqrt{1 + 16y^2}dy$$

making the substitution $u = 1 + 16y^2, \quad ydy = \frac{du}{32}$ we get

$$2\pi \int_{17}^{65} \frac{\sqrt{u}}{32}du$$

$$= \frac{\pi}{16}[\frac{2}{3}((65)^{3/2} - (17)^{3/2})]$$

Second way: as an "x" integral:

$$2\pi \int_3^9 y\sqrt{1 + (\frac{dy}{dx})^2}dx$$

Differentiating $x = 1 + 2y^2$ we get $4y\frac{dy}{dx} = 1$ so the integral becomes

$$2\pi \int_3^9 y\sqrt{1 + (\frac{1}{4y})^2}dx$$

$$= 2\pi \int_3^9 \sqrt{y^2 + \frac{1}{16}}dx$$

substituting for y^2 we get

$$2\pi \int_3^9 \sqrt{\frac{x-1}{2} + \frac{1}{16}}dx$$

$$= 2\pi \int_3^9 \frac{1}{4}\sqrt{8x - 7}dx$$

putting $u = 8x - 7$ we get

$$2\pi \int_{17}^{65} \frac{1}{32}\sqrt{u}du$$

AS BEFORE!!