# Practice final, Math 1A, Fall 2005
# V. Jones

YOUR NAME:

YOUR TA's NAME:

YOUR SECTION NUMBER:

There are two parts.

**PART I** is multiple choice and contains 6 problems. Each of these problems is worth 2 points for a total of 12 points possible. No partial credit.

**PART II** has 15 problems. You must **show** your **work** to get full credit on these problems.

**No calculators, or books but a two-sided "cheat-sheet page" is allowed.**

Do all work on the exam, use back pages if you need more space.

PART I:

PART II:

TOTAL:

**Circle the MOST CORRECT answers for PART I**

## PART I

(multiple choice)

1. Which of the following is correct for positive $x, y$ and $a$?

   (a) $\log_a(x+y) = \log_a(x)\log_a(y)$.

   (b) $2\log_a(x) = \log_a(x^2)$.

   (c) $(\log_a(x))^2 = \log_a(2x)$.

   (d) $a^x = e^{a \ln x}$.

   (e) $e^x = a^{e \ln x}$.

   (f) $e^{\log_a(x)} = a^{x \ln e}$.

2. Which of the following is a correct definition of $\lim_{x\to\infty} f(x) = L$?.

   (a) For all $\epsilon \geq 0$ there is a $\delta > 0$ such that $x > -\delta \implies |f(x) - L| < \epsilon$.

   (b) For all $\epsilon > 0$ there is a $\delta > 0$ such that $x > \delta \implies |f(x) - L| < \epsilon$.

   (c) For all $\epsilon \geq 0$ there is a $\delta > 0$ such that $f(x) > \delta \implies |f(x) - L| \leq \epsilon$.

   (d) For all $\epsilon > 0$ there is a $\delta > 0$ such that $x < -\delta \implies |f(x) - L| \leq \epsilon$.

   (e) For all $\epsilon > 0$ there is a $\delta > 0$ such that $f(x) > \frac{1}{\delta} \implies |f(x) - L| \leq \epsilon$.

   (f) For all $\epsilon \geq 0$ there is a $\delta > 0$ such that $x > \frac{1}{\delta} \implies |f(x) - L| \leq \epsilon$.

3. Which of the following is correct concerning the function $f(x) = \frac{1}{x}\cos\frac{1}{x}$ defined for all real $x$ besides 0 which is NOT IN THE DOMAIN OF THE FUNCTION.

(a) $f(x)$ is continuous at 0 by the squeeze theorem.

(b) $f(x)$ is discontinuous at all points in the domain of $f$.

(c) $f(x)$ is continuous at 0 because $|x|$ is continuous.

(d) $f(x)$ is continous at 0 because it is differentiable at 0.

(e) $\lim_{x\to 0^+} = \infty$.

(f) $f(x)$ has neither a left nor a right limit as $x \to 0^{\pm}$

4. Suppose that the domain of $f(x)$ is $(0, \infty)$ and that for $x \neq 4$,

$$f(x) = x^2 + 32\frac{x-4}{\sqrt{x}-2}.$$

Suppose $f'(4)$ exists. Find $f''(4)$.

(a) There is not enough information to solve for $f''(4)$

(b) 2

(c) 1

(d) $-2$

(e) $-1$

(f) 4

5. Which of the following is a Riemann sum for $\int_1^2 x^2 dx$?

(a) $\sum_{i=1}^{n} 1 + (\frac{i}{n})^2$

(b) $\frac{1}{n}\sum_{i=1}^{n} 1 + (\frac{i}{n})^2$

(c) $n\sum_{i=1}^{n} 1 + (\frac{i}{n})^2$

(d) $\sum_{i=1}^{n}(1 + \frac{i}{n})^2$

(e) $n\sum_{i=1}^{n}(1+\frac{i}{n})^2$

(f) $\frac{1}{n}\sum_{i=1}^{n}(1+\frac{i}{n})^2$

6. If $f(x)$ and $g(x)$ are continuous functions on $[a,b]$ and $a<c<b$, which of the following is necessarily true?

   (a) If $f(x)>g(x)$ for $a<x<c$ and $f(x)<g(x)$ for $c<x<b$ then $\int_a^c f(x)dx \quad > \int_c^b g(x)dx$

   (b) If $f(x)>g(x)$ for $a<x<c$ and $f(x)<g(x)$ for $c<x<b$ then $\int_a^c (f(x)-g(x))dx \quad > \int_c^b (g(x)-f(x))dx$

   (c) If $f(x)>g(x)$ for $a<x<c$ and $f(x)<g(x)$ for $c<x<b$ then $\int_a^c |f(x)|dx \quad > \int_c^b |g(x)|dx$

   (d) If $f(x)>g(x)$ for $a<x<c$ and $f(x)<g(x)$ for $c<x<b$ then $\int_a^c (f(x)-g(x))dx \quad > \int_c^b (f(x)-g(x))dx$

   (e) If $f(x)>g(x)$ for $a<x<c$ and $f(x)<g(x)$ for $c<x<b$ then $\int_a^c f(x)^2dx \quad > \int_c^b f(x)dx$

   (f) If $f(x)>g(x)$ for $a<x<c$ and $f(x)<g(x)$ for $c<x<b$ then $\int_a^c f(x)^2dx \quad > \int_c^b g(x)dx$

## PART II

1. (3 points - each answer right or wrong-no partial credit) Differentiate:

   (a) $f(x)=x^3-4x+7$

   (b) $f(x)=\int_0^x \sqrt{\cos(t^3)} \quad dt$

(c) $f(x) = x(\ln x)(e^x)$

2. (3 points) (i) True or false
If $f(x)$ is continuous and satisfies $f(x) = -f(-x)$ for $-a \leq x \leq a$ then $\int_{-a}^{a} f(x)dx = 0$.

(ii) True or false:
If $f(x)$ is differentiable at $x$ then

$$\lim_{h \to 0} \frac{f(x + \sin h) - f(x)}{h} = f'(x)$$

. (iii) True or false: The fundamental theorem of calculus asserts that a function that is differentiable on an interval has a local maximum on that interval.

3. (6 points-2 each) Evaluate the following indefinite integrals:

(a) $\displaystyle\int \frac{\sin x}{1 - \cos^2 x} dx$

(b) $\displaystyle\int \frac{e^x}{e^x + 1} dx$

(c) $\displaystyle\int \cosh x \quad dx$

4. (6 points-2 each) Evaluate the following definite integrals.

(a) $\displaystyle\int_0^1 x^3 \quad dx$

(b) $\displaystyle\int_{-1}^1 x e^{x^4} \quad dx$

(c) $\displaystyle\int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$

5. (3 points)Evaluate the following limits:
(i) $\displaystyle\lim_{x \to 0} \frac{\tan x}{\sec x}$

(ii) $\lim_{x\to\infty} (1+2/x)^x$

(iii) $\lim_{t\to 0} \dfrac{e^{3t}-1}{t}$

6. (3 points) Sketch the graph of $y = \dfrac{x^2}{x-4}$

7. (3 points) Evaluate the limit:

$$\lim_{n\to\infty} \frac{1}{n}\Big[\big(\frac{1}{n}\big)^9 + \big(\frac{2}{n}\big)^9 + \big(\frac{3}{n}\big)^9 + \cdots + \big(\frac{n}{n}\big)^9\Big]$$

8. (3 points) Express $\int_2^4 \sin x \quad dx$ as a limit of Riemann sums.

9. (3 points) For what values of $x$ is $f(x) = x|x|$ differentiable? Find a formula for the derivative.

10. (3 points) Differentiate with respect to $x$: $\displaystyle\int_{x^2}^{cosx} cos2t \quad dt$

11. (3 points) Find the area of the bounded region bounded by the curves $x = 2y^2$ and $x + y = 1$.

12. (3 points) Find the volume of the solid formed when the region between $x = y^2$ and $y = x^2$ is rotated about the line $x = -1$.

13. (3points) Explain what the fundamental theorem of calculus is and why it is true.

14. (3 points) Use cylindrical shells to find the volume of the region bounded by $x = 1 + y^2$, $x = 0$, $y = 1$ and $y = 2$ about the $x$ axis. Make sure to sketch the region and a typical shell.

15. (3 points) Find the average of the function $\dfrac{x}{\sqrt{1+2x}}$ over the interval $[0, 4]$.