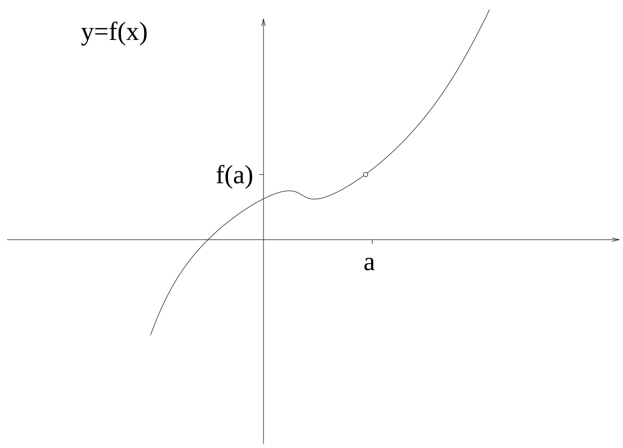
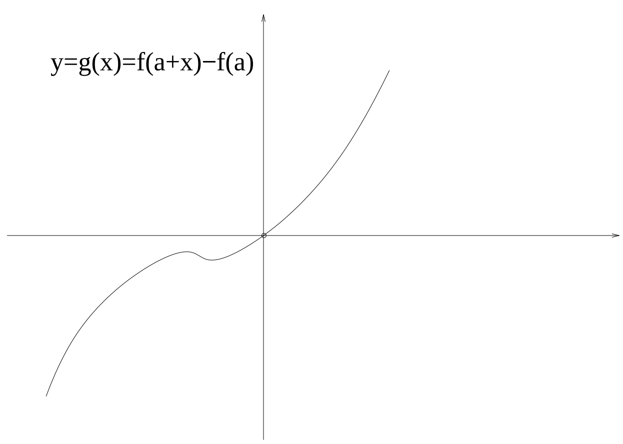
Here is an alternative way of visualising the derivative of a function $f(x)$ at a point $x = a$. The idea is simple-if we zoom in on the graph of a function it should look more and more like a straight line. The slope of the limiting straight line is called the derivative $f'(a)$ of f at a.

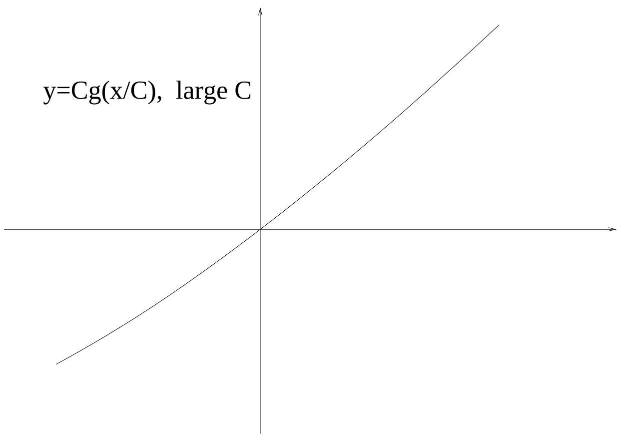
So start with a function $f(x)$ whose graph is depicted below:



Scaling f is most conveniently done at the origin so move the graph so that the point $(a, f(a))$ becomes the origin - i.e. consider $g(x)$ depicted below:



Now zoom in on the picture at the origin with a large scaling factor C:



Recall that if $y = h(x)$ is a straight line through the origin, the slope of that line is $h(1)$. So the slope of the straight line to which the graph of g is getting closer at the origin is approximately

$$Cg(\frac{1}{C})$$

or, in terms of f,

$$C\{f(a + \frac{1}{C}) - f(a)\}$$

The above approximation to the slope of the limiting straight line should be as good as C is large so we deduce:

$$f'(a) = \lim_{C\to\infty} C\{f(a + \frac{1}{C}) - f(a)\}$$

To reconcile this definition with the usual one put $1/C = \delta$ so $C = \frac{1}{\delta}$ and the formula becomes:

$$f'(a) = \lim_{\delta\to 0} \frac{f(a+\delta) - f(a)}{\delta}$$