Homework Due Thursday 21 April

In these questions remember that you may only assume things we have proved in class. Pay attention to the details!

1)Let c and s be differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ satisfying $c(0) = 1$,$s(0) = 0$,$s'(x) = c(x) \quad \forall x$ and $c'(x) = -s(x) \quad \forall x$. Show that

$$c(x) = \cos x, \quad s(x) = \sin(x) \quad \forall x.$$

2) Show that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

3) Show that $\cos t = 0$ for some unique t between 1 and 2 and $\cos x > 0$ for $x \in (-t, t)$.

Define the real number π by $\pi = 2t$ where t is as in question 2.

4) Show that $\cos(\pi/2 - x) = \sin x$, $\sin(\pi/2 - x)$, $\sin(x + \pi) = -\sin x$ and $\cos(x + \pi) = -\cos x$.

5) Show $\exp(x)$ is a bijective function from $\mathbb{R}$ to $(0, \infty)$. Let $\log x$ be the inverse function to exp. Show that log is differentiable and $\log' x = \frac{1}{x}$.

6) Show that for $-1 < x < 1$,

$$\log(1 - x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

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7) Let $f : [0, 2] \to \mathbb{R}$ be continuous and differentiable in $(0, 2)$ and $f(0) = 0$. Suppose $-3 < f'(x) < 4$ for $x \in (0, 2)$. Show

$$-6 < f(2) < 8$$

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