Homework due Thursday 17 March 2011

1) Let $f : X \to Y$ be a continuous function from one metric space to another. Show that if X is compact, f is uniformly continuous.

2) Let (X, d) be a compact metric space and $X^{\mathbb{N}}$ be the set of all sequences in X. Show that $D((x_n), (y_n)) = \sum_{n=1}^{\infty} \frac{d(x_n, y_n)}{2^n}$ defines a metric on $X^{\mathbb{N}}$ for which it is compact. (Hint:use sequential compactness.)

3) The four color theorem asserts that any finite map in the plane can be colored with 4 colors (so that no adjacent countries have the same color). Use this to show that any planar map at all (i.e. possibly countably infinite) can be colored with four colors.

4) Show that a compact metric space is complete.

5) 22.3 on page 168.

6) 22.5 on page 169