Let S be a set, a relation on S is a subset of $S \times S$ and we write for $s, t \in S$, $s \sim t$ when (s, t) is in that subset.

An *equivalence relation* is a relation $\sim$ with the three properties
(i) $s \sim s \quad \forall s \in S$
(ii) $s \sim t \iff t \sim s$
(iii) If $s \sim t$ and $t \sim r$ then $s \sim r$.

A *partition* of S is a collection $\{S_i | i \in I\}$ of subsets of S with

$$S = \coprod_{i \in I} S_i$$

.

The notion of equivalence relation and partition are actually the same.
Given an equivalence relation $\sim$ and an element $s \in S$ we write $[s]$ for $\{t \in S | t \sim s\}$ and call it the *equivalence class* of s. It is easy to check the following:
(i) $s \in [s]$ so $S = \cup_{s \in S}[s]$
(ii) $[s] = [t] \iff s \sim t$ and $[s] \cap [t] = \phi$ otherwise.

So S is the disjoint union of the equivalence classes.

On the other hand if a partition $\coprod_i S_i$ of S is given we define $\sim$ by
$(s \sim t) \iff \exists i$ such that $s \in S_i$ and $t \in S_i$.

It is easy to check that $\sim$ is an equivalence relation whose equivalence classes are the S_i.