Math 128a, Section 3 — Problem Set 10 — Due Nov 27, 2001

(1) Problem 8.1.2.

(2) Problem 8.1.5.

(3) Problem 8.1.12. There are several typos in this problem: it should use an approximation of the form

$$R(x) = a \pm \frac{1}{x - b}$$

which matches f and f' at $x = c$. The $\pm$ sign will be chosen to match the sign of $f'(c)$, so the approximation is increasing or decreasing in accord with f.

(4) Let's apply the analytical technique of p. 278 for the square root iteration to the problem of computing inverses (for division). (a) Fix $a > 0$ and apply Newton's method to $f(x) = a - 1/x$ to derive the iteration

$$x_{n+1} = 2x_n - ax_n^2.$$

(b) Fix $a = 2$ and show that the iteration of (a) converges to $x = 1/2$ for sufficiently close starting values like $0 < x < 1$. (c) Fix an invertible square matrix

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

and try the matrix equivalent

$$X_{n+1} = 2X_n - X_nAX_n, \qquad X_0 = I/2.$$

How many steps does it take before the residual $\|AX - I\|_1$ becomes less than the machine precision? Is the convergence quadratic? Why or why not?