Math 202B **Spring 2004**
Sarason

HOMEWORK ASSIGNMENT 7

Due in class on Friday, March 12.

25. Let f be in $L^p(\lambda_N)$ and g in $L^{p'}(\lambda_N)$. Prove that $f * g$ is bounded and continuous.

26. (a) Let the function $\psi : \mathbb{R} \to \mathbb{R}$ be of class C^∞ and have compact support. Let the function g be in $L^p(\lambda)$ $(1 \leq p \leq \infty)$. Prove that $\psi * g$ is of class C^∞, with $(\psi * g)^{(n)} = \psi^{(n)} * g$ for all positive integers n.

    (b) Use Part (a) to prove that $C^\infty(\mathbb{R}) \cap L^p(\lambda)$ is dense in $L^p(\lambda)$ for $1 \leq p < \infty$.

27. Let E be a Lebesgue measurable subset of $\mathbb{R}^N$ of finite positive measure. Prove that the set

$$E - E = \{x - y : x, y \in E\}$$

    contains a neighborhood of the origin.

28. Prove that the algebra $L^1(\lambda)$ has no identity (λ = Lebesgue measure on $\mathbb{R}$).