Math 202B **Spring 2004**
Sarason

HOMEWORK ASSIGNMENT 6

Due in class on Friday, March 5.

21. Suppose the sequence $(f_n)_1^\infty$ in $L^1(\mu)$ converges almost everywhere to the function f in $L^1(\mu)$, and that $\lim_{n\to\infty} \|f_n\|_1 = \|f\|_1$. Prove that $\lim_{n\to\infty} \|f - f_n\|_1 = 0$.

22. Let $(X, \mathcal{R}, \mu)$ and $(Y, \mathcal{S}, \nu)$ be σ-finite measure spaces. Let A be a set in the hereditary σ-ring generated by $\mathcal{R}$ and B a set in the hereditary σ-ring generated by $\mathcal{S}$. Prove that $(\mu \times \nu)^*(A \times B) = \mu^*(A)\nu^*(B)$.

23. (a) Prove that if f is in $L^1(\mu)$ then $\lim_{t\to\infty} t\Lambda_f(t) = 0$.

    (b) Prove that if μ is a finite measure, if f is a measurable function, and if there is a positive number ϵ such that $\Lambda_f(t) = O(1/t(\log t)^{1+\epsilon})$ as $t \to \infty$, then f is in $L^1(\mu)$.

24. Let f be in $L^1(\lambda)$. Define the function g on $\mathbb{R}$ by

$$g(x) = \int_{\mathbb{R}} \frac{f(y)}{1 + x^2 + y^2} dy.$$

    Prove that g is in $L^1(\lambda)$, and that $\|g\|_1 \le \pi \|f\|_1$.