Math 202B **Spring 2004**
Sarason

HOMEWORK ASSIGNMENT 5

Due in class on Friday, February 27.

17. Let f and g be in $L^p(\mu)$, where $1 < p < \infty$. Prove that the inequality $\|f + g\|_p \leq \|f\|_p + \|g\|_p$ is strict unless $f = 0$ a.e., or $g = 0$ a.e., or g is a positive scalar multiple of f. (Geometric interpretation: The closed unit ball of $L^p(\mu)$ is strictly convex.)

18. Assume $1 \leq p_0 < p_1 < \infty$. For $0 < t < 1$ define p_t by

$$\frac{1}{p_t} = \frac{1-t}{p_0} + \frac{t}{p_1}.$$

Prove that if f is in $L^{p_0}(\mu) \cap L^{p_1}(\mu)$, then f is in $L^{p_t}(\mu)$ and

$$\|f\|_{p_t} \leq \|f\|_{p_0}^{1-t} \|f\|_{p_1}^{t}.$$

19. Let f be in $L^\infty(\mu)$ and in $L^p(\mu)$ for some finite p. Prove that $\|f\|_\infty = \lim_{p\to\infty} \|f\|_p$.

20. Let λ be Lebesgue measure on $\mathbb{R}$. Let p be in $(1, \infty)$. Construct a function that is in $L^p(\lambda)$ but that fails to be in $L^q(\lambda)$ for q in $[1, \infty)\setminus\{p\}$.