Math 202B **Spring 2004**
Sarason

HOMEWORK ASSIGNMENT 4

Due in class on Friday, February 20.

13. Let $(X, \mathcal{A}, \mu)$ be a finite measure space and let f be a nonnegative measurable function on X. Prove that f is integrable if and only if

$$\sum_{n=1}^{\infty} \mu(\{f > n\}) < \infty.$$

14. For α a real number, define the function f_α on $\mathbb{R}$ by $f_\alpha(x) = |x|^{2\alpha}/(1 + x^2)$. Prove that f is Lebesgue integrable if and only if $-\frac{1}{2} < \alpha < \frac{1}{2}$.

15. Let f be a Lebesgue-integrable function on $\mathbb{R}$. Prove that the series

$$\sum_{n=-\infty}^{\infty} f(x+n)$$

converges absolutely for almost every x in $\mathbb{R}$.

16. Let f be a Lebesgue-integrable function on $\mathbb{R}^N$. For $r \geq 0$ let $B_r = \{x \in \mathbb{R}^N : \|x\| \leq r\}$, and define the function $g : [0, \infty) \to \mathbb{R}$ by

$$g(r) = \int_{B_r} f \, d\lambda_N$$

(λ_N = Lebesgue measure). Prove g is continuous.