Math 202B **Spring 2004**
Sarason

HOMEWORK ASSIGNMENT 3

Due in class on Friday, February 13.

9. Let E be a Lebesgue null subset of $\mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function. Prove $f(E)$ is a null set.

10. Let X be a set, $\mathcal{A}$ a σ-algebra on X, and $(f_n)_1^\infty$ a sequence of real-valued $\mathcal{A}$-measurable functions. Prove that the set of points where $\lim_{n\to\infty} f_n$ exists finitely belongs to $\mathcal{A}$.

11. Let X be a topological space and $\mathcal{F}$ a family of continuous real-valued functions on X. Prove that the function g defined by

$$g(x) = \sup\{f(x) : f \in \mathcal{F}\}$$

is Borel measurable. (Note that $\mathcal{F}$ need not be countable.)

12. Let X be a set and $\mathcal{A}$ a σ-algebra on X. A complex-valued function f on X is said to be $\mathcal{A}$-measurable if its real and imaginary parts are $\mathcal{A}$-measurable. Prove that this happens if and only if $f^{-1}(B)$ is in $\mathcal{A}$ for every Borel subset B of the complex plane.