Math 202B **Spring 2004**
Sarason

HOMEWORK ASSIGNMENT 2

Due in class on Friday, February 6.

5. Let μ be a finite measure on an algebra $\mathcal{A}$ of subsets of a set X. Prove that a subset E of X is μ^*-measurable if and only if $\mu^*(E) + \mu^*(X \backslash E) = \mu(X)$. (The finiteness assumption on μ is just the assumption that $\mu(X) < \infty$.)

6. Let μ be a measure on a ring $\mathcal{R}$ and let E be a set in the hereditary σ-ring generated by $\mathcal{R}$. Prove that for E to be μ^*-measurable it is sufficient that

$$\mu(A) = \mu^*(A \cap E) + \mu^*(A \backslash E)$$

for every A in $\mathcal{R}$ such that $\mu(A) < \infty$.

7. Construct a meager subset of $\mathbb{R}^N$ whose complement has Lebesgue measure 0.

8. For $S \subset \mathbb{R}$, define $\mu^*(S)$ to be 0 if S is countable, 1 if S is uncountable but there is a countable set C such that $S \backslash C$ is bounded, and ∞ otherwise. Prove that μ^* is an outer measure, and describe the μ^*-measurable sets. Is every μ^*-measurable set σ-finite? Can μ^* be induced by a measure on a ring?