

## HOMework ASSIGNMENT 12

Due in class on Friday, April 23.

46. Let  $B_1$  and  $B_2$  be Banach spaces, and let the transformation  $T$  in  $\mathcal{L}(B_1, B_2)$  have a closed range. Prove  $T^*$  has a closed range.
47. Let  $B_1$  and  $B_2$  be Banach spaces, and let the transformation  $T$  in  $\mathcal{L}(B_1, B_2)$  have the property that  $TB_1$  has a finite codimension in  $B_2$  (i.e.,  $B_2/TB_1$  is finite dimensional). Prove  $TB_1$  is closed.
48. Prove that, in  $C[0, 1]$ , the functions that are differentiable at  $1/2$  form a meager set.
49. Prove that, for  $0 < p < 1$ , there are no nontrivial continuous linear functionals on  $L^p(0, 1)$ .