Math 110, Section 7 **Fall 2003**
Sarason

SUGGESTED EXERCISES

Pages 244–246, Exercises 3, 4, 6, 7, 8, 9, 11, 14, 15, 16, 19, plus Exercises 1–4 below. In Exercises 1–3 below, V is a complex vector space of dimension 4 and T is in $\mathcal{L}(V)$.

1. In each part, assuming the given matrix is the Jordan matrix for T, write down the minimal polynomial q_T and the characteristic polynomial p_T of T.

$$\text{(a)} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(d)} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. (a) Suppose T has characteristic polynomial $p_T(z) = (z^2 - 1)^2$. Write down all possibilities for the Jordan matrix for T.

   (b) Suppose T has minimal polynomial $q_T(z) = (z-1)(z^2-1)$. Write down all possibilities for the Jordan matrix for T.

3. In each part, suppose T has the given matrix with respect to some basis. Find the Jordan matrix for T.

$$\text{(a)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{(d)} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

4. Let T_1 and T_2 be nilpotent operators on the vector space V of dimension n. Prove that T_1 and T_2 are similar if and only if $\dim(\text{null } T_1^k) = \dim(\text{null } T_2^k)$ for $k = 1, \ldots, n-1$