Math 104, Section 6 **Fall 2003**
Sarason

HOMEWORK ASSIGNMENT 9

Due in class on Wednesday, November 5.

Exercise 17.14 on page 125, Exercise 21.8 on page 163, plus Exercises AA–DD below.

AA. Find a one-to-one continuous function of $\mathbb{R}$ onto $(0, 1)$.

BB. Let X, Y, Z be metric spaces. Let the functions $f : X \to Y$ and $g : Y \to Z$ be uniformly continuous. Prove $g \circ f$ is uniformly continuous.

CC. Let (X, d) be a metric space. Regard $X \times X$ as a metric space with the metric e defined by

$$e((x_1, y_1), (x_2, y_2)) = d(x_1, x_2) + d(y_1, y_2).$$

Prove the function $d : X \times X \to \mathbb{R}$ is uniformly continuous.

DD. Let (X, d) be a metric space. Define $e : X \times X \to \mathbb{R}$ by $e(x, y) = d(x, y)/(1 + d(x, y))$. Prove that e is a metric, that the identity map of (X, d) to (X, e) is uniformly continuous, and that the identity map of (X, e) to (X, d) is uniformly continuous.