Math 104, Section 6 **Fall 2003**
Sarason

HOMEWORK ASSIGNMENT 7

Due in class on Wednesday, October 22.

P. Prove that the following sets are open.

(a) $U_1 = \{x \in \mathbb{R}^k : \|x\| > 1\}$
(b) $U_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 > 0\}$
(c) $U_3 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 > x_4, x_2 < x_3\}$

Q. Prove that the following sets are closed.

(a) $F_1 = \{x \in \mathbb{R}^k : \|x\| = 1\}$
(b) $F_2 = \{(x, y) \in \mathbb{R}^2 : y \geq x^2\}$
(c) $F_3 = \{(x, y, z) \in \mathbb{R}^3 : xyz = 0\}$

R. Find the set of limit points of the subset $A = \left\{\left(\frac{1}{n}, \frac{k}{n}\right) : n \in \mathbb{N}, k \in \mathbb{N}_n\right\}$ of $\mathbb{R}^2$.

S. Let $\mathcal{B}_0$ be the family of balls $B_r(q)$ in $\mathbb{R}^k$ such that q is in $\mathbb{Q}^k$ and r is in $\mathbb{Q}$.

(a) Prove $\mathcal{B}_0$ is countable.
(b) Prove that, if U is an open subset of $\mathbb{R}^k$, then U is the union of the balls in $\mathcal{B}_0$ that it contains.

T. Let F be an infinite closed subset of $\mathbb{R}^k$. Prove there is a countable set whose closure is F.

U. (a) Let F be a closed subset of $\mathbb{R}^k$. Prove there are open sets $U_1, U_2, \ldots$ such that $F = \bigcap_{n=1}^{\infty} U_n$.

(b) Let U be an open subset of $\mathbb{R}^k$. Prove there are closed sets $F_1, F_2, \ldots$ such that $U = \bigcup_{n=1}^{\infty} F_n$.