Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 8

Due in class on Wednesday, November 3.

32. Prove that the function $f(x) = \sqrt{x}$ $(0 \le x < \infty)$ is uniformly continuous.

33. Let the sequence $(f_n)_1^\infty$ of uniformly continuous functions from the metric space M to the metric space N converge uniformly to the function f. Prove f is uniformly continuous.

34. Let M, N, P be metric spaces and $f : M \to N$, $g : N \to P$ uniformly continuous functions. Prove the composite function $g \circ f$ is uniformly continuous.

35. Let (M, d) be a metric space. Make $M \times M$ into a metric space in the usual way. Prove the function $d : M \times M \to \mathbb{R}$ is uniformly continuous.

36. Consider the metric space $C[0, 1]$. A function h in $C[0, 1]$ is called piecewise linear if $[0, 1]$ can be partitioned by means of finitely many points $a_0, a_1, \ldots, a_n$, called the nodes of h, such that $0 = a_0 < a_1 < \cdots < a_n = 1$, and such that h is linear on each subinterval $[a_{j-1}, a_j]$, meaning that for $j = 1, \ldots, n$,

$$h((1-t)a_{j-1} + ta_j) = (1-t)h(a_{j-1}) + th(a_j), \quad 0 \le t \le 1.$$

Let $\mathcal{S}$ be the set of piecewise linear functions in $C[0, 1]$ having rational nodes and taking rational values at the nodes. Prove $\mathcal{S}$ is dense in $C[0, 1]$. Deduce that $C[0, 1]$ is separable. (Suggestion: Try workng with equally spaced nodes.)