Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 7

Due in class on Wednesday, October 27.

27. Let $(a_n)_1^\infty$ be a bounded sequence in $\mathbb{R}$, with $\alpha = \liminf_{n\to\infty} a_n$ and $\beta = \limsup_{n\to\infty} a_n$. Prove $\limsup_{n\to\infty} a_n^2 = \max\{\alpha^2, \beta^2\}$.

28. Le $(a_n)_1^\infty$ be a convergent sequence and $(b_n)_1^\infty$ a bounded sequence in $\mathbb{R}$. Prove

$$\limsup_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \limsup_{n\to\infty} b_n.$$

29. Prove the set $\{x \in \mathbb{R}^3 : \|x\|_2 = 1\}$, the 2-sphere, is connected.

30. Let A and B be nonempty subsets of $\mathbb{R}^M$ and $\mathbb{R}^N$, respectively. Prove $A \times B$ is a connected subset of $\mathbb{R}^{M+N}$ if and only if A and B are connected. (Here, we are making the natural identification between $\mathbb{R}^M \times \mathbb{R}^N$ and $\mathbb{R}^{M+N}$.)

31. Let $(K_n)_1^\infty$ be a nested sequence of nonempty, compact, connected subsets of a metric space M. Prove the set $K = \bigcap_{n=1}^\infty K_n$ is connected. (Suggestion: Argue by contradiction.)