Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 5

Due in class on Wednesday, October 6.

17. Prove that every nonempty open subset of $\mathbb{R}^N$ can be expressed as a union of open balls with rational centers and rational radii. (Note: A point of $\mathbb{R}^N$ is called rational if it is in $\mathbb{Q}^N$.)

18. Find a continuous function $f : \mathbb{R} \to \mathbb{R}$ whose range is neither open nor closed.

19. Let $(x_n)_1^\infty$ be a sequence in the metric space (M, d). The point x_0 of M is called a cluster point of the sequence $(x_n)_1^\infty$ if, for every $\epsilon > 0$, the set $\{n \in \mathbb{N} : d(x_0, x_n) < \epsilon\}$ is infinite. Prove x_0 is a cluster point of $(x_n)_1^\infty$ if and only if there is a subsequence of $(x_n)_1^\infty$ that converges to x_0.

20. Let C be the set of cluster points of the sequence $(x_n)_1^\infty$ in the metric space (M, d). Prove C is a closed set.

21. Let G be an open subset of a metric space. Prove $\operatorname{int}(\partial G) = \emptyset$.