Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 3

Due in class on Wednesday, September 22.

8. Let $(a_n)_1^\infty$ be a convergent sequence of positive real numbers with limit b. Prove that $\lim_{n\to\infty}\sqrt{a_n} = \sqrt{b}$. (Suggestion: Treat separately the cases $b = 0$ and $b > 0$.)

9. (a) Let $(a_n)_1^\infty$ be a convergent sequence of real numbers with limit b. Define the sequence $(b_n)_1^\infty$ by $b_n = \frac{1}{n}(a_1 + a_2 + \cdots + a_n)$. Prove that $\lim_{n\to\infty} b_n = b$. (Suggestion: Reduce the general case to the case $b = 0$.)

   (b) Find a divergent sequence $(a_n)_1^\infty$ such that the sequence $(b_n)_1^\infty$ defined as in part (a) is convergent.

10. Let t be a number in the interval $(0, 1)$. Define the sequence $(a_n)_1^\infty$ by $a_1 = 1$, $a_{n+1} = t(a_n + 1)$.

    (a) Prove the sequence $(a_n)_1^\infty$ bounded and monotone.

    (b) Find $\lim_{n\to\infty} a_n$.

11. Let $F_1, F_2, \ldots$ be the Fibonacci numbers, defined by $F_1 = F_2 = 1$, $F_{n+2} = F_n + F_{n+1}$. Let $r_n = F_{n+1}/F_n$.

    (a) Prove the sequences $(r_{2n})_1^\infty$ and $(r_{2n-1})_1^\infty$ are monotone.

    (b) Prove $(r_n)_1^\infty$ converges, and find its limit.