Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 2

Due in class on Wednesday, September 15.

4. Prove that every subfield of $\mathbb{R}$ contains $\mathbb{Q}$.

5. Let A and B be nonempty subsets of $\mathbb{R}$ such that

   (a) $A \cup B = \mathbb{R}$;

   (b) if a is in A and b is in B then $a < b$;

   (c) A contains no largest element.

   Prove B contains a smallest element.

6. Let x be a positive real number and n a natural number. Prove there are rational numbers a and b such that
$$x - \frac{1}{n} < a^2 < x < b^2 < x + \frac{1}{n}.$$

7. Let $A_1, A_2, \dots$ be nonempty subsets of $\mathbb{R}$ such that the set $A = \bigcup_{n=1}^{\infty} A_n$ is bounded above. For each n let $b_n = \sup A_n$, and let $B = \{b_1, b_2, \dots\}$. Prove that $\sup A = \sup B$.