Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 10

Due in class on Wednesday, November 24.

41. Let $f : [a, b] \to \mathbb{R}$ be bounded and let $g : [a, b] \to \mathbb{R}$ be Riemann integrable. Prove
$$U(f + g) = U(f) + U(g).$$

42. Let $f : [a, b] \to \mathbb{R}$ be bounded and Riemann integrable over $[a + \epsilon, b]$ for every ϵ in $(0, b - a)$. Prove f is Riemann integrable over $[a, b]$.

43. By a step function on $[a, b]$ is meant a finite linear combination of characteristic functions of subintervals of $[a, b]$. Prove such a function is Riemann integrable.

44. Prove that the characteristic function of the Cantor set is Riemann integrable over $[0, 1]$ and that its integral is 0.

45. Construct a Riemann integrable function on $[0, 1]$ that is discontinuous at each point of $\mathbb{Q} \cap (0, 1)$. (Suggestion: To guarantee integrability, make the function nondecreasing.)