Math 104, Section 1 **Fall 2004**
Sarason

HOMEWORK ASSIGNMENT 1

Due in class on Wednesday, September 8.

1. Prove that the number of subsets of the set $\mathbb{N}_n = \{1, 2, \ldots, n\}$ is 2^n.

2. Prove that, for m and n natural numbers, the number of ordered m-tuples whose coordinates belong to $\mathbb{N}_n$ equals n^m. (Suggestion: Use induction on m.)

3. For $n = 0, 1, 2, \ldots$ and $k = 0, 1, \ldots, n$, the binomial coefficient $\binom{n}{k}$ is defined by $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. (Recall the convention $0! = 1$.)

   (a) Establish the identity $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$.

   (b) Prove the binomial theorem:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k, \quad n = 1, 2, \ldots \ .$$