M. Rieffel Math 202A, Midterm Exam
October 27, 2014

SHOW YOUR WORK COMPLETELY AND NEATLY.
DON'T WRITE MORE THAN YOU NEED TO.
Total points = 30.

Name ....

SID# ....

Scores:

problem 1 ....

problem 2 ....

problem 3 ....

1. a) (2 points) Define what is meant by a *compact* topological space.

b) (8 points) Let X be a compact topological space, let Y be a Hausdorff topological space, and let f be a continuous function from X to Y. Prove that if f is bijective (i.e. one-to-one and onto), then f is a homeomorphism.

2. a) (2 points) Let X and Y be two topological spaces. Define what is meant by the *product topology* on $X \times Y$.

b) (8 points) Let $\mathbb{R}$ have its usual metric topology, and let $\mathcal{T}_P$ be the product topology on $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$. Let $\mathcal{T}_M$ be the metric topology from the usual Euclidean metric on $\mathbb{R}^2$. Prove that $\mathcal{T}_P = \mathcal{T}_M$.

3. a) (2 points) Let $(X, \mathcal{T})$ be a topological space, let Y be a set, and let π be a function from X onto Y. Define what is meant by the corresponding *quotient topology* on Y.

b) (8 points) Consider the equivalence relation on the unit interval $[0, 1]$ in $\mathbb{R}$ given as follows: $0 \sim t$ only if $t = 0$, and $1 \sim t$ only if $t = 1$, and $s \sim t$ for all s and t in the open interval $(0, 1)$. Let Y be the set of equivalence classes for this equivalence relation. Let $[0, 1]$ have its usual metric topology. List all of the open sets for the corresponding quotient topology on Y. Is this quotient topology Hausdorff? Why?