M. Rieffel Math 202A, Homework 14

due Friday November 30, 2018

1. Prove that

$$\lim_{a\to+\infty}\int_0^a (\sin(t))/t\ dt$$

exists and is finite, but that $t \to (\sin(t))/t$ is not Lebesgue integrable.

2. Let f be defined and Lebesgue intergrable on the interval $[0,+\infty)$. Define g on that interval by

$$g(t) = \int_0^\infty e^{-tx} f(x)\ dx.$$

Prove that g is differentiable for $t > 0$, and that

$$g'(t) = -\int_0^\infty e^{-tx} x f(x)\ dx$$

for $t > 0$.

3. a) Let f be the function defined on $\mathbb{R}$ by $f(t) = t^{-1/2}$ for t in the interval $(0,1)$, and $f(t) = 0$ for t not in that interval. Prove that f is Lebesgue integrable.

b) Let $\{r_n\}_{n=1}^\infty$ be an enumeration of the rational numbers. For f defined as in part a), define g a.e. on $\mathbb{R}$ by

$$g(t) = \sum_{n=1}^\infty 2^{-n} f(t - r_n).$$

Prove that the series defining g converges a.e., and that g is Lebesgue integrable. But show that g is unbounded on any open interval of $\mathbb{R}$, and in fact that any function that agrees with g a.e. is unbounded on any open interval

c) Calculate $\int_{-\infty}^\infty g(t)\ dt$.