M. Rieffel Math 202A, Homework 10 due November 2, 2018

1. a) Let f be a continuous function from $\mathbb{R}$ to a Banach space B. Prove that f is Borel-measurable, i.e. is S-measurable (in the sense defined in class) where S is the σ-field of all Borel subsets of $\mathbb{R}$, that is, the σ-field generated by the open (or by the compact) subsets of $\mathbb{R}$.

b) Describe a natural class of topological spaces X for which you can prove the same result as in part a) but with $\mathbb{R}$ replaced by X (with essentially the same proof).

2. Let X be an uncountable set, equipped with the discrete topology, and let S be the σ-field (generated by the open subsets of X) consisting of all subsets of X. Let $B = \ell^\infty(X)$, the Banach space of all bounded $\mathbb{R}$-valued functions on X with the uniform (i.e. supremum) norm. For each $x \in X$ let δ_x be the function on X taking value 1 at x and value 0 at all other points. So each δ_x is an element of B. Define a continuous function, f, from X to B by $f(x) = \delta_x$ for all $x \in X$. Is f an S-measurable function? Why?