

Homework, week 7

1. (BC, p. 286), 1(for grade), 2, 3 (hint: start with the second integral), 5 (for grade).
2. (BC, p. 296), 1, 6 (for grade).
3. (for grade) Verify that $f(z) = z^k$ is a one-to-one analytic mapping in a neighborhood of any $z_0 \neq 0$ ($k \neq 0$ is an integer).
4. (a) Prove that a linear function maps polygon to a polygon.
(b) Prove that if f is an entire function and for some rectangle R , the image $f(R)$ is also a rectangle, then f is linear.
5. (for grade) Describe the image of the upper half plane under a mapping of the form

$$f(z) = \frac{az + b}{cz + d}$$

where a, b, c, d are real and $ad - bc < 0$.

6. Find a formula for all conformal automorphisms of the upper half plane.