

Find the region of convergence of the following series:

1. (for grade)

$$\sum_{n=1}^{\infty} (n + (-1)^n 2^n) z^n$$

- 2.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} z^n$$

- 3.

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} z^n$$

The geometric series: $\sum_{n=1}^{\infty} z^n = \frac{1}{1-z}$ converges absolutely for $|z| < 1$.

1. Find the region of convergence of

$$\sum_{n=1}^{\infty} n^2 z^n$$

,

2. (for grade) Find explicitly the function to which this power series is convergent in the region of convergence.
3. Describe the region where this function is defined.

Evaluate the integral $\int_C f dz$, when :

1. $f(z) = x^2 + iy^2$, $C = \{z(t) = t^2 + it | 0 \leq t \leq 1\}$,
2. (for grade) $f(z) = \frac{1}{z}$, $C = \{z(t) = \sin t + i \cos t | 0 \leq t \leq 2\pi\}$,
3. $f(z) = \frac{1}{z+2}$, $C = \{z(t) = \cos t + i \sin t | 0 \leq t \leq 2\pi\}$,
4. (for grade) $f(z) = \frac{1}{z(z+1)(z+2)}$, $C = \{z(t) = t + 1 | 0 \leq t < \infty\}$ (improper integral).