Practice problems for the Midterm 2.

1. Find singularities of the function and describe their type:

$$f(z) = \frac{\sin(\frac{1}{z})}{(\pi z)^2 - 1}, \quad \frac{1}{\cos(z) - 1 - z^2/2}$$

$$f(z) = f(z) = \begin{cases} \frac{\sin(z)}{z(z+\pi)(z-1)} & , \quad z \neq 0, 1, -\pi \\ 1 & , \quad z = 0 \\ -\frac{1}{\pi(\pi+1)} & , \quad z = -\pi \end{cases}$$

2. Find Laurent series for

$$\frac{z}{z^4+1}, \text{ about } z = 0, \quad \frac{1}{(z-1)(z^2-1)}, \text{ about } z = 1,$$

$$\frac{\sin(z^2)}{z^3}, \text{ about } z = 0, \quad \frac{\sin(\frac{\pi z}{2})}{1+z}, \text{ about } z = -1,$$

3. Find residues of functions at all their singular points

$$(z+1)\cos(\frac{1}{z}), \quad \frac{\sin^2(\frac{z}{2})}{1-\cos z}, \quad \frac{z+1}{(z-1)^3 z}$$

$$ze^{\frac{1}{z}}, \quad (z+z^2)e^{\frac{1}{z}}, \quad e^{z+\frac{1}{z}}$$

4. Compute the following improper integrals using the residues

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2+1} dx, \quad \int_{-\infty}^{\infty} \frac{x \sin x}{(x^2+1)^2} dx,$$

$$p.v. \int_{-\infty}^{\infty} \frac{1}{(x^3-1)} dx, \quad p.v. \int_{-\infty}^{\infty} \frac{\sin x}{x(x^2+1})dx$$

5. Let C be the counter clock wise oriented circle around the origin of radius 2. For each function f indicate if it is true or false that $\int_C f(z)dz = 0$.

$$f(z) = \frac{1}{z(z-1)}, \quad f(z) = \sin z, \quad f(z) = \tan(10z), \quad f(z) = \frac{1}{z^2+5}$$

6. Find the decomposition into simple fractions for

$$\frac{1}{z^2(z-1)^2}, \quad \frac{z}{z^3(z-2)}$$