

Practice problems for the Final exam.

1. Answer yes or no to the following question. Does the function f has a limit as $z \rightarrow 0$?

(a) $f = \sqrt{\frac{z}{\bar{z}}}$

(b) $f = z \sin(\frac{1}{z})$

(c) $f = \frac{\sin z}{z}$

(d) $f = z\bar{z}$

2. Find Laurent series for

$$\frac{1}{z(z^3 + 1)}, \text{ about } z = 0, \quad \frac{1}{(z - 1)^2(z - 2)}, \text{ about } z = 1,$$

$$\frac{1 - \cos(z^2)}{z^3}, \text{ about } z = 0, \quad \frac{\sinh(z)}{z^2}, \text{ about } z = 0,$$

3. Find residues of functions at all their singular points

$$(z^2 + 1) \sin\left(\frac{1}{z}\right), \quad \frac{1}{1 - z \cos z}, \text{ (only at } z = 0) \quad \frac{z^2 + 1}{(z - 1)^2 z}$$

$$z^3 e^{\frac{1}{z}}, \quad (z + z^{-1}) e^{\frac{1}{z}}, \quad e^{z^2 + \frac{1}{z^2}}$$

4. Is it true that if $f = u + iv$ is an entire function such $|u| < M$, and $|v| < N$ for some M, N and all z , then $f'(z) = 0$.

5. Is the function

$$f(z) = \begin{cases} \frac{\sin(z)}{z} & , \quad z \neq 0 \\ 1 & , \quad z = 0 \end{cases}$$

entire? (just indicate "yes" or "no", you do not have to show your work)

6. Is the function $f(z) = \operatorname{Re}(\cos z) + \operatorname{Im}(\cos z)$ analytic? (just indicate "yes" or "no", you do not have to show your work)

7. $f(\frac{1}{z})$ is analytic for all $z \neq 0$ and $\lim_{z \rightarrow 0} f(\frac{1}{z}) = 2$. Find $f(1)$.

8. Using residues compute the integrals

$$\int_0^\infty \frac{x^\alpha}{(x+2)(x+1)} dx, \quad \int_1^\infty \frac{\sqrt{x-1}}{(x+1)x} dx,$$

9. Identify each sketch of a vector field with one of the functions: a) $\frac{1}{z}$, b) \sqrt{z} , c) $-\sqrt{z}$, d) $-i\frac{1}{z}$, e) $i\bar{z}$. Some functions may not match any pictures.