Math 16B, Section 1 **Spring 2006**
Sarason

ASSIGNMENT 1

Exercise 7.1.7. Let a rectangular box have sides of length x, y, z, measured in feet, with z being the height. Suppose the material used for the top and bottom of the box costs \$3 per square foot and the material for the sides costs \$5 per square foot. Find the formula for the function $C(x, y, z)$ expressing the total cost of the material in the box.

Solution. The top and bottom each has area xy, so the cost for the material in each is $3xy$. There are two sides of area xz, each contributing $5xz$ to the cost. There are two sides of area yz, each contributing $5yz$ to the cost. The total cost is therefore given by

$$C(x, y, z) = 6xy + 10xz + 10yz \text{ dollars.}$$

Exercise 7.1.8. A rectangular enclosure is open at the front and has no floor. Its height is z, its front and back have length x, and its two sides have length y, all measured in feet. The material for its top costs \$3 per square foot and the material for the back and two sides costs \$5 per square foot. Find a formula for the function $C(x, y, z)$ expressing the total cost of the material in the enclosure.

Solution. This is very much like the preceding exercise, the difference being that now the area costing \$3 per square foot is xy (instead of $2xy$), and the area costing \$5 per square foot is $xz + 2yz$ (instead of $2xz + 2yz$). The total cost is therefore given by

$$C(x, y, z) = 3xy + 5xz + 10yz.$$

Exercise 7.1.12. In a certain manufacturing process, labor costs \$100 per unit and capital costs \$200 per unit. Find a formula for the function $C(x, y)$ expressing the cost incurred when using x units of labor and y units of capital.

Solution. The total cost for labor will be $100x$ dollars and for capital it will be $200y$ dollars. Hence

$$C(x, y) = 100x + 200y \text{ dollars.}$$

Exercise 7.1.13. A property in Lalaville has a market value of v dollars. For real estate tax purposes, its assessed value is $.40v$, and there is a homeowners exemption of x dollars, giving a net assessed value of $.40v - x$. The tax rate is r dollars per \$100 of net assessed value, giving a total tax of

$$T = f(r, v, x) = \frac{r}{100}(.40v - x).$$

(a) If the market value is \$200,000, the homeowners exemption is \$5000, and the tax rate is $r = \$2.50$, what is the tax?

Solution. This is plug-and-chug. We have $v = 200,000$, $x = 5000$, $r = 2.50$, so

$$\begin{aligned} T &= f(2.5,\ 200,000,\ 5000) = \frac{2.5}{100}((.4)(200,000) - 5000) \\ &= .025(80,000 - 5000) = .025(75,000) \\ &= 1875 \text{ dollars (ugh!)} \end{aligned}$$

(b) Suppose the tax rate r is increased by 20% to \$3.00, but v and x are unchanged. What is the new tax? Is it 20% higher than its former value?

Solution. Since r appears only as a factor in the formula for the function $f(r, v, x)$, an increase in r, the other variables being held fixed, will result in a proportionate increase in T. Hence, the new tax will equal 1.2 times the old tax:

$$T_{\text{new}} = (1.2)(1875) = 2250 \text{ dollars (double ugh!)}$$

Exercise 7.2.2. Find the first partial derivatives of the function $f(x, y) = 3x^2 + 2y + 1$.

Solution.

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial(3x^2)}{\partial x} + \frac{\partial(2y)}{\partial x} + \frac{\partial(1)}{\partial x} = 6x + 0 + 0 = 6x \\ \frac{\partial f}{\partial y} &= \frac{\partial(3x^2)}{\partial y} + \frac{\partial(2y)}{\partial y} + \frac{\partial(1)}{\partial y} = 0 + 2 + 0 = 2. \end{aligned}$$

Exercise 7.2.5. Find the first partial derivatives of the function $f(x, y) = y^2/x$.

Solution.

$$\begin{aligned} \frac{\partial}{\partial x}(y^2/x) &= y^2\frac{\partial(1/x)}{\partial x} = -y^2/x^2 \\ \frac{\partial}{\partial y}(y^2/x) &= \frac{1}{x}\frac{\partial(y^2)}{\partial y} = 2y/x. \end{aligned}$$

Exercise 7.2.6. Find the first partial derivatives of the function $f(x, y) = x/(1 + e^y)$.

Solution.

$$\begin{aligned} \frac{\partial}{\partial x}\left(\frac{x}{1+e^y}\right) &= \frac{1}{1+e^y}\frac{\partial x}{\partial x} = \frac{1}{1+e^y} \\ \frac{\partial}{\partial y}\left(\frac{x}{1+e^y}\right) &= x\frac{\partial}{\partial y}\left(\frac{1}{1+e^y}\right) = \frac{-xe^y}{(1+e^y)^2}. \end{aligned}$$

In the last step, the chain rule was used, in the form

$$\frac{d}{dy}\left(\frac{1}{g(y)}\right) = \frac{-g'(y)}{g(y)^2}.$$

Exercise 7.2.17. Find the first partial derivative of the function $f(x, y, z) = xze^{yz}$.

Solution.

$$\begin{aligned}
\frac{\partial}{\partial x}(xze^{yz}) &= ze^{yz} \\
\frac{\partial}{\partial y}(xze^{yz}) &= xz\frac{\partial}{\partial y}(e^{yz}) = xz^2e^{yz} \\
\frac{\partial}{\partial z}(xze^{yz}) &= xz\frac{\partial(e^{yz})}{\partial z} + e^{yz}\frac{\partial(xz)}{\partial z} \\
&= xyze^{yz} + xe^{yz}.
\end{aligned}$$