

# Representation Theory, Geometry & Combinatorics Seminar

Organizer(s): M. Haiman, K. Reshetikhin, D. Hill & J. Sussan

Monday, 1:00–3:00pm, 939 Evans

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11/17/2008 **Luca Scala**, University of Chicago

*Strange duality on the projective plane and cohomology of tautological bundles on Hilbert schemes of points on a surface*

Let  $M_n$  the moduli space of semistable sheaves on the complex projective plane  $\mathbb{P}_2$  of rank 2, degree 0 and second Chern class  $n$ . The Barth morphism associates to the class  $[F] \in M_n$  of a semistable sheaf  $F$  its curve of jumping lines  $\gamma(F) \in |\mathcal{O}_{\mathbb{P}_2^*}(n)|$  in  $\mathbb{P}_2^*$ , that is, the lines  $\ell \in \mathbb{P}_2^*$ , such that  $F|_\ell$  is not trivial. Consider the determinant line bundle  $\mathcal{D}_n := \gamma^* \mathcal{O}_{|\mathcal{O}_{\mathbb{P}_2^*}(n)|}(1)$  on  $M_n$ . The morphism  $\gamma^* : H^0(\mathbb{P}_2^*, \mathcal{O}_{\mathbb{P}_2^*}(n))^* \longrightarrow H^0(M_n, \mathcal{D}_n)$ , induced on the level of global sections of line bundles  $\mathcal{O}_{\mathbb{P}_2^*}(n)$  and  $\mathcal{D}_n$ , is conjectured to be an isomorphism by Beauville and can be interpreted as an example of strange morphism for the moduli spaces  $M_n$  and  $\mathbb{P}_2^*$ ; it fits in the general strange duality conjecture on smooth complex projective surfaces formulated by Le Potier. The technique used to attack the conjecture in this example, due to Le Potier, He and Danila, employs moduli spaces of coherent systems in order to interpolate the moduli space  $M_n$  with a Hilbert scheme  $\mathbb{P}_2^{[m]}$  of  $m$  points in  $\mathbb{P}_2$ ; this procedure allows to reduce the computation of the space of global sections of the determinant line bundle on  $M_n$  to the study of the cohomology of the Hilbert scheme  $H^p(\mathbb{P}_2^{[m]}, S^q L^{[m]})$ , where  $L^{[m]}$  is the tautological bundle on  $\mathbb{P}_2^{[m]}$  associated to a certain line bundle  $L$  on  $\mathbb{P}_2$ . Danila's results on the cohomology of  $S^q L^{[m]}$  yield the strange duality for  $0 \leq n \leq 19$ .

We will show how to improve and generalize Danila's results on the cohomology of the Hilbert scheme of points on a surface  $X$  with values in representations of tautological bundles. By making use of the derived McKay correspondence of Bridgeland-King-Reid, adapted by Haiman in the case of the Hilbert scheme  $X^{[m]}$ , we prove general formulas for the cohomology of  $X^{[m]}$  with values in the double tensor power  $L^{[m]} \otimes L^{[m]}$  and general exterior powers  $\Lambda^k L^{[m]}$  of tautological bundles.