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Math54 Final Exam, Fall 2007

This is a closed everything exam, except a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	16	
3	16	
4	16	
5	16	
6	16	
7	16	
Total	100	

1. (5 Points) Write your personal information below.

   Your Name: ____________________________

   Your GSI: ____________________________

   Your SID: ____________________________

2. Find bases for the image and kernel for

$$A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

3. Let

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}.$$

   (a) Find the eigenvalues and corresponding eigenvectors of A.

   (b) Diagonalize A.

4. Consider the parameterized differential equation

$$x'' + 4x' + \alpha x = 0,$$

where α is a real constant.

(a) Find all the roots of the auxilary equation. Discuss for what values of α are the roots real, complex, or multiple.

(b) In each case find the general solution to the differential equation.

5. Let u and v be vectors in $\mathcal{R}^n$. Form the matrix

$$G = \begin{pmatrix} \begin{pmatrix} u^T u \\ v^T u \end{pmatrix} & \begin{pmatrix} u^T v \\ v^T v \end{pmatrix} \end{pmatrix}.$$

Show that u and v are linearly independent if and only if G is invertible.

6. Let q_1, $q_2, ..., q_k$ be a set of orthonormal vectors in $\mathcal{R}^n$. Sow that there exists vectors q_{k+1}, $q_{k+2}, ..., q_n$ so that

$$Q = (q_1 \quad q_2 \cdots \quad q_n)$$

is an orthonormal matrix.

7. Given a function $f(x) = \cos\left(\sqrt{2}x\right)$ on the interval $(0, \pi)$.

   (a) Extend it into an odd function on $(-\pi, \pi)$.

   (b) Compute the Fourier sine series of the function on the interval $(0, \pi)$.