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Math54H Final Exam, Fall 2013

This is a closed everything exam, except a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Your Name: ______________________________

Your SID: ______________________________

1. (a) Find bases for the column and null spaces of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

(b) Let

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}.$$

i. Find the eigenvalues and corresponding eigenvectors of A.

ii. Diagonalize A.

2. Consider the parameterized differential equation

$$x'' + 4x' + \alpha x = 0,$$

where α is a real constant.

(a) Find all the roots of the auxilary equation. Discuss for what values of α are the roots real, complex, or multiple.

(b) In each case find the general solution to the differential equation.

3. Let q_1, $q_2, ..., q_k$ be a set of orthonormal vectors in $\mathcal{R}^n$. Sow that there exists vectors q_{k+1}, $q_{k+2}, ..., q_n$ so that

$$Q = (q_1 \quad q_2 \cdots \quad q_n)$$

is an orthonormal matrix.

4. Given a function $f(x) = \cos\left(\sqrt{2}x\right)$ on the interval $(0, \pi)$.

   (a) Extend it into an odd function on $(-\pi, \pi)$.

   (b) Compute the Fourier sine series of the function on the interval $(0, \pi)$.

5. Final all solutions of the form $u(x,t) = X(x)T(t)$ for the following equations

$$\begin{aligned}\frac{\partial u(x,t)}{\partial t} &= \frac{\partial^2 u(x,t)}{\partial x^2}, \\ u(0,t) &= 0, \quad \frac{\partial u(L,t)}{\partial x} = 0.\end{aligned}$$