

44 Linear Algebra and Differential Equations

32. Could a set of three vectors in \mathbb{R}^4 span all of \mathbb{R}^4 ? Explain. What about n vectors in \mathbb{R}^m when n is less than m ?
33. Suppose A is a 4×3 matrix and \mathbf{b} is a vector in \mathbb{R}^4 with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. What can you say about the reduced echelon form of A ? Justify your answer.
34. Let A be a 3×4 matrix, let \mathbf{v}_1 and \mathbf{v}_2 be vectors in \mathbb{R}^3 , and let $\mathbf{w} = \mathbf{v}_1 + \mathbf{v}_2$. Suppose $\mathbf{v}_1 = A\mathbf{u}_1$ and $\mathbf{v}_2 = A\mathbf{u}_2$ for some vectors \mathbf{u}_1 and \mathbf{u}_2 in \mathbb{R}^4 . What fact allows you to conclude that the system $A\mathbf{x} = \mathbf{w}$ is consistent? (Note: \mathbf{u}_1 and \mathbf{u}_2 denote vectors, not scalar entries in vectors.)
35. Let A be a 5×3 matrix, let \mathbf{y} be a vector in \mathbb{R}^3 , and let \mathbf{z} be a vector in \mathbb{R}^5 . Suppose $A\mathbf{y} = \mathbf{z}$. What fact allows you to conclude that the system $A\mathbf{x} = 5\mathbf{z}$ is consistent?
36. Suppose A is a 4×4 matrix and \mathbf{b} is a vector in \mathbb{R}^4 with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Explain why the columns of A must span \mathbb{R}^4 .

[M] In Exercises 37–40, determine if the columns of the matrix span \mathbb{R}^4 .

$$37. \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \quad 38. \begin{bmatrix} 4 & -5 & -1 & 8 \\ 3 & -7 & -4 & 2 \\ 5 & -6 & -1 & 4 \\ 9 & 1 & 10 & 7 \end{bmatrix}$$

$$39. \begin{bmatrix} 10 & -7 & 1 & 4 & 6 \\ -8 & 4 & -6 & -10 & -3 \\ -7 & 11 & -5 & -1 & -8 \\ 3 & -1 & 10 & 12 & 12 \end{bmatrix}$$

$$40. \begin{bmatrix} 5 & 11 & -6 & -7 & 12 \\ -7 & -3 & -4 & 6 & -9 \\ 11 & 5 & 6 & -9 & -3 \\ -3 & 4 & -7 & 2 & 7 \end{bmatrix}$$

41. [M] Find a column of the matrix in Exercise 39 that can be deleted and yet have the remaining matrix columns still span \mathbb{R}^4 .
42. [M] Find a column of the matrix in Exercise 40 that can be deleted and yet have the remaining matrix columns still span \mathbb{R}^4 . Can you delete more than one column?

SG Mastering Linear Algebra Concepts: Span 1–18

WEB

SOLUTIONS TO PRACTICE PROBLEMS

1. The matrix equation

$$\begin{bmatrix} 1 & 5 & -2 & 0 \\ -3 & 1 & 9 & -5 \\ 4 & -8 & -1 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} -7 \\ 9 \\ 0 \end{bmatrix}$$

is equivalent to the vector equation

$$3 \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix} - 2 \begin{bmatrix} 5 \\ 1 \\ -8 \end{bmatrix} + 0 \begin{bmatrix} -2 \\ 9 \\ -1 \end{bmatrix} - 4 \begin{bmatrix} 0 \\ -5 \\ 7 \end{bmatrix} = \begin{bmatrix} -7 \\ 9 \\ 0 \end{bmatrix}$$

which expresses \mathbf{b} as a linear combination of the columns of A .

$$2. \quad \mathbf{u} + \mathbf{v} = \begin{bmatrix} 4 \\ -1 \end{bmatrix} + \begin{bmatrix} -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$A(\mathbf{u} + \mathbf{v}) = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 + 20 \\ 3 + 4 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \end{bmatrix}$$

$$\begin{aligned} A\mathbf{u} + A\mathbf{v} &= \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ 11 \end{bmatrix} + \begin{bmatrix} 19 \\ -4 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \end{bmatrix} \end{aligned}$$