

1.3 EXERCISES

In Exercises 1 and 2, compute $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - 2\mathbf{v}$.

1. $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$ 2. $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

In Exercises 3 and 4, display the following vectors using arrows on an xy -graph: $\mathbf{u}, \mathbf{v}, -\mathbf{v}, -2\mathbf{v}, \mathbf{u} + \mathbf{v}, \mathbf{u} - \mathbf{v}$, and $\mathbf{u} - 2\mathbf{v}$. Notice that $\mathbf{u} - \mathbf{v}$ is the vertex of a parallelogram whose other vertices are $\mathbf{u}, \mathbf{0}$, and $-\mathbf{v}$.

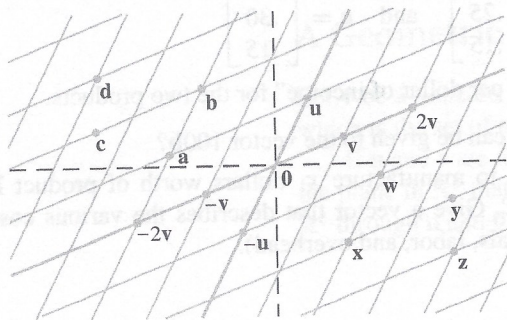
3. \mathbf{u} and \mathbf{v} as in Exercise 1 4. \mathbf{u} and \mathbf{v} as in Exercise 2

In Exercises 5 and 6, write a system of equations that is equivalent to the given vector equation.

5. $x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ -9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$

6. $x_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Use the accompanying figure to write each vector listed in Exercises 7 and 8 as a linear combination of \mathbf{u} and \mathbf{v} . Is every vector in \mathbb{R}^2 a linear combination of \mathbf{u} and \mathbf{v} ?



7. Vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and \mathbf{d}

8. Vectors $\mathbf{w}, \mathbf{x}, \mathbf{y}$, and \mathbf{z}

In Exercises 9 and 10, write a vector equation that is equivalent to the given system of equations.

9. $x_2 + 5x_3 = 0$ 10. $3x_1 - 2x_2 + 4x_3 = 3$
 $4x_1 + 6x_2 - x_3 = 0$ $-2x_1 - 7x_2 + 5x_3 = 1$
 $-x_1 + 3x_2 - 8x_3 = 0$ $5x_1 + 4x_2 - 3x_3 = 2$

In Exercises 11 and 12, determine if \mathbf{b} is a linear combination of $\mathbf{a}_1, \mathbf{a}_2$, and \mathbf{a}_3 .

11. $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

12. $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$

In Exercises 13 and 14, determine if \mathbf{b} is a linear combination of the vectors formed from the columns of the matrix A .

13. $A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$

14. $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

15. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$. For what value(s) of h is \mathbf{b} in the plane spanned by \mathbf{a}_1 and \mathbf{a}_2 ?

16. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$. For what value(s) of h is \mathbf{y} in the plane generated by \mathbf{v}_1 and \mathbf{v}_2 ?

In Exercises 17 and 18, list five vectors in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. For each vector, show the weights on \mathbf{v}_1 and \mathbf{v}_2 used to generate the vector and list the three entries of the vector. Do not make a sketch.

17. $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$

18. $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

19. Give a geometric description of $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ for the vectors $\mathbf{v}_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}$.

20. Give a geometric description of $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ for the vectors in Exercise 18.

21. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ for all h and k .

22. Construct a 3×3 matrix A , with nonzero entries, and a vector \mathbf{b} in \mathbb{R}^3 such that \mathbf{b} is not in the set spanned by the columns of A .

In Exercises 23 and 24, mark each statement True or False. Justify each answer.

23. a. Another notation for the vector $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ is $[-4 \ 3]$.

- b. The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.

- c. An example of a linear combination of vectors \mathbf{v}_1 and \mathbf{v}_2 is the vector $\frac{1}{2}\mathbf{v}_1$.

- d. The solution set of the linear system whose augmented matrix is $[a_1 \ a_2 \ a_3 \ b]$ is the same as the solution set of the equation $x_1 a_1 + x_2 a_2 + x_3 a_3 = b$.
- e. The set $\text{Span}\{u, v\}$ is always visualized as a plane through the origin.
24. a. When u and v are nonzero vectors, $\text{Span}\{u, v\}$ contains only the line through u and the origin, and the line through v and the origin.
- b. Any list of five real numbers is a vector in \mathbb{R}^5 .
- c. Asking whether the linear system corresponding to an augmented matrix $[a_1 \ a_2 \ a_3 \ b]$ has a solution amounts to asking whether b is in $\text{Span}\{a_1, a_2, a_3\}$.
- d. The vector v results when a vector $u - v$ is added to the vector v .
- e. The weights c_1, \dots, c_p in a linear combination $c_1 v_1 + \dots + c_p v_p$ cannot all be zero.
25. Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by a_1, a_2, a_3 , and let $W = \text{Span}\{a_1, a_2, a_3\}$.
- a. Is b in $\{a_1, a_2, a_3\}$? How many vectors are in $\{a_1, a_2, a_3\}$?
- b. Is b in W ? How many vectors are in W ?
- c. Show that a_1 is in W . [Hint: Row operations are unnecessary.]
26. Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $b = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A .
- a. Is b in W ?
- b. Show that the second column of A is in W .
27. A mining company has two mines. One day's operation at mine #1 produces ore that contains 30 metric tons of copper and 600 kilograms of silver, while one day's operation at mine #2 produces ore that contains 40 metric tons of copper and 380 kilograms of silver. Let $v_1 = \begin{bmatrix} 30 \\ 600 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 40 \\ 380 \end{bmatrix}$. Then v_1 and v_2 represent the "output per day" of mine #1 and mine #2, respectively.
- a. What physical interpretation can be given to the vector $5v_1$?
- b. Suppose the company operates mine #1 for x_1 days and mine #2 for x_2 days. Write a vector equation whose solution gives the number of days each mine should operate in order to produce 240 tons of copper and 2824 kilograms of silver. Do not solve the equation.
- c. [M] Solve the equation in (b).
28. A steam plant burns two types of coal: anthracite (A) and bituminous (B). For each ton of A burned, the plant produces 27.6 million Btu of heat, 3100 grams (g) of sulfur dioxide, and 250 g of particulate matter (solid-particle pollutants). For

each ton of B burned, the plant produces 30.2 million Btu, 6400 g of sulfur dioxide, and 360 g of particulate matter.

- a. How much heat does the steam plant produce when it burns x_1 tons of A and x_2 tons of B?
- b. Suppose the output of the steam plant is described by a vector that lists the amounts of heat, sulfur dioxide, and particulate matter. Express this output as a linear combination of two vectors, assuming that the plant burns x_1 tons of A and x_2 tons of B.
- c. [M] Over a certain time period, the steam plant produced 162 million Btu of heat, 23,610 g of sulfur dioxide, and 1623 g of particulate matter. Determine how many tons of each type of coal the steam plant must have burned. Include a vector equation as part of your solution.
29. Let v_1, \dots, v_k be points in \mathbb{R}^3 and suppose that for $j = 1, \dots, k$ an object with mass m_j is located at point v_j . Physicists call such objects *point masses*. The total mass of the system of point masses is

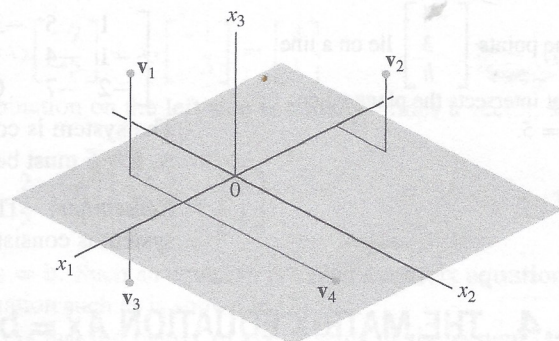
$$m = m_1 + \dots + m_k$$

The *center of gravity* (or *center of mass*) of the system is

$$\bar{v} = \frac{1}{m} [m_1 v_1 + \dots + m_k v_k]$$

Compute the center of gravity of the system consisting of the following point masses (see the figure):

Point	Mass
$v_1 = (2, -2, 4)$	4 g
$v_2 = (-4, 2, 3)$	2 g
$v_3 = (4, 0, -2)$	3 g
$v_4 = (1, -6, 0)$	5 g



30. Let v be the center of mass of a system of point masses located at v_1, \dots, v_k as in Exercise 29. Is v in $\text{Span}\{v_1, \dots, v_k\}$? Explain.

If statement (d) is true, then each row of U contains a pivot position and there can be no pivot in the augmented column. So $A\mathbf{x} = \mathbf{b}$ has a solution for any \mathbf{b} , and (a) is true. If (d) is false, the last row of U is all zeros. Let \mathbf{d} be any vector with a 1 in its last entry. Then $[U \ \mathbf{d}]$ represents an *inconsistent* system. Since row operations are reversible, $[U \ \mathbf{d}]$ can be transformed into the form $[A \ \mathbf{b}]$. The new system $A\mathbf{x} = \mathbf{b}$ is also inconsistent, and (a) is false.

PRACTICE PROBLEMS

- Let $A = \begin{bmatrix} 1 & 5 & -2 & 0 \\ -3 & 1 & 9 & -5 \\ 4 & -8 & -1 & 7 \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \\ 0 \\ -4 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -7 \\ 9 \\ 0 \end{bmatrix}$. It can be shown that \mathbf{p} is a solution of $A\mathbf{x} = \mathbf{b}$. Use this fact to exhibit \mathbf{b} as a specific linear combination of the columns of A .
- Let $A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$. Verify Theorem 5(a) in this case by computing $A(\mathbf{u} + \mathbf{v})$ and $A\mathbf{u} + A\mathbf{v}$.

1.4 EXERCISES

Compute the products in Exercises 1–4 using (a) the definition, as in Example 1, and (b) the row–vector rule for computing $A\mathbf{x}$. If a product is undefined, explain why.

- $\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 3 & -4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

In Exercises 5–8, use the definition of $A\mathbf{x}$ to write the matrix equation as a vector equation, or vice versa.

- $\begin{bmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$
- $\begin{bmatrix} 2 & -3 \\ 3 & 2 \\ 8 & -5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -21 \\ 1 \\ -49 \\ 11 \end{bmatrix}$
- $x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$
- $z_1 \begin{bmatrix} 2 \\ -4 \end{bmatrix} + z_2 \begin{bmatrix} -1 \\ 5 \end{bmatrix} + z_3 \begin{bmatrix} -4 \\ 3 \end{bmatrix} + z_4 \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

In Exercises 9 and 10, write the system first as a vector equation and then as a matrix equation.

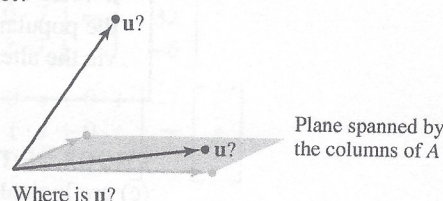
- $5x_1 + x_2 - 3x_3 = 8$
 $2x_2 + 4x_3 = 0$
- $4x_1 - x_2 = 8$
 $5x_1 + 3x_2 = 2$
 $3x_1 - x_2 = 1$

Given A and \mathbf{b} in Exercises 11 and 12, write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

$$11. A = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & 2 \\ -3 & -7 & 6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ 4 \\ 12 \end{bmatrix}$$

$$12. A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

- Let $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is \mathbf{u} in the plane in \mathbb{R}^3 spanned by the columns of A ? (See the figure.) Why or why not?



- Let $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. Is \mathbf{u} in the subset of \mathbb{R}^3 spanned by the columns of A ? Why or why not?

15. Let $A = \begin{bmatrix} 3 & -1 \\ -9 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible \mathbf{b} , and describe the set of all \mathbf{b} for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

16. Repeat the requests from Exercise 15 with

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Exercises 17–20 refer to the matrices A and B below. Make appropriate calculations that justify your answers and mention an appropriate theorem.

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$$

17. How many rows of A contain a pivot position? Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each \mathbf{b} in \mathbb{R}^4 ?
18. Can every vector in \mathbb{R}^4 be written as a linear combination of the columns of the matrix B above? Do the columns of B span \mathbb{R}^4 ?
19. Can each vector in \mathbb{R}^4 be written as a linear combination of the columns of the matrix A above? Do the columns of A span \mathbb{R}^4 ?
20. Do the columns of B span \mathbb{R}^4 ? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for each \mathbf{y} in \mathbb{R}^4 ?

21. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$. Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span \mathbb{R}^4 ? Why or why not?

22. Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 9 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 4 \\ -2 \\ -6 \end{bmatrix}$. Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span \mathbb{R}^3 ? Why or why not?

In Exercises 23 and 24, mark each statement True or False. Justify each answer.

23. a. The equation $A\mathbf{x} = \mathbf{b}$ is referred to as a *vector equation*.
 b. A vector \mathbf{b} is a linear combination of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.
 c. The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $[A \ \mathbf{b}]$ has a pivot position in every row.
 d. The first entry in the product $A\mathbf{x}$ is a sum of products.
 e. If the columns of an $m \times n$ matrix A span \mathbb{R}^m , then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each \mathbf{b} in \mathbb{R}^m .
 f. If A is an $m \times n$ matrix and if the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some \mathbf{b} in \mathbb{R}^m , then A cannot have a pivot position in every row.

24. a. Every matrix equation $A\mathbf{x} = \mathbf{b}$ corresponds to a vector equation with the same solution set.
 b. If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then \mathbf{b} is in the set spanned by the columns of A .
 c. Any linear combination of vectors can always be written in the form $A\mathbf{x}$ for a suitable matrix A and vector \mathbf{x} .
 d. If the coefficient matrix A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent.
 e. The solution set of a linear system whose augmented matrix is $[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{b}]$ is the same as the solution set of $A\mathbf{x} = \mathbf{b}$, if $A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3]$.
 f. If A is an $m \times n$ matrix whose columns do not span \mathbb{R}^m , then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^m .

25. Note that $\begin{bmatrix} 4 & -3 & 1 \\ 5 & -2 & 5 \\ -6 & 2 & -3 \end{bmatrix} \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix}$. Use this

fact (and no row operations) to find scalars c_1, c_2, c_3 such

$$\text{that } \begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix} = c_1 \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}.$$

26. Let $\mathbf{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$. It can be shown that $2\mathbf{u} - 3\mathbf{v} - \mathbf{w} = \mathbf{0}$. Use this fact (and no row operations) to find x_1 and x_2 that satisfy the equation

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}.$$

27. Rewrite the (numerical) matrix equation below in symbolic form as a vector equation, using symbols $\mathbf{v}_1, \mathbf{v}_2, \dots$ for the vectors and c_1, c_2, \dots for scalars. Define what each symbol represents, using the data given in the matrix equation.

$$\begin{bmatrix} -3 & 5 & -4 & 9 & 7 \\ 5 & 8 & 1 & -2 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ -11 \end{bmatrix}$$

28. Let $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$, and \mathbf{v} represent vectors in \mathbb{R}^5 , and let x_1, x_2 , and x_3 denote scalars. Write the following vector equation as a matrix equation. Identify any symbols you choose to use.
 $x_1\mathbf{q}_1 + x_2\mathbf{q}_2 + x_3\mathbf{q}_3 = \mathbf{v}$
29. Construct a 3×3 matrix, not in echelon form, whose columns span \mathbb{R}^3 . Show that the matrix you construct has the desired property.
30. Construct a 3×3 matrix, not in echelon form, whose columns do not span \mathbb{R}^3 . Show that the matrix you construct has the desired property.
31. Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all \mathbf{b} in \mathbb{R}^3 . Generalize your argument to the case of an arbitrary A with more rows than columns.