

## 24 Linear Algebra and Differential Equations

2. a.  $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  b.  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

c.  $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d.  $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Row reduce the matrices in Exercises 3 and 4 to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

3.  $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$  4.  $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$

5. Describe the possible echelon forms of a nonzero  $2 \times 2$  matrix. Use the symbols  $\blacksquare$ ,  $*$ , and  $0$ , as in the first part of Example 1.

6. Repeat Exercise 5 for a nonzero  $3 \times 2$  matrix.

Find the general solutions of the systems whose augmented matrices are given in Exercises 7–14.

7.  $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$  8.  $\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix}$

9.  $\begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -3 & 4 & -6 \end{bmatrix}$  10.  $\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$

11.  $\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$  12.  $\begin{bmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

13.  $\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

14.  $\begin{bmatrix} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Exercises 15 and 16 use the notation of Example 1 for matrices in echelon form. Suppose each matrix represents the augmented matrix for a system of linear equations. In each case, determine if the system is consistent. If the system is consistent, determine if the solution is unique.

15. a.  $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b.  $\begin{bmatrix} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & 0 \end{bmatrix}$

16. a.  $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \end{bmatrix}$

b.  $\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$

In Exercises 17 and 18, determine the value(s) of  $h$  such that the matrix is the augmented matrix of a consistent linear system.

17.  $\begin{bmatrix} 1 & -1 & 4 \\ -2 & 3 & h \end{bmatrix}$  18.  $\begin{bmatrix} 1 & -3 & 1 \\ h & 6 & -2 \end{bmatrix}$

In Exercises 19 and 20, choose  $h$  and  $k$  such that the system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

19.  $\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$  20.  $\begin{aligned} x_1 - 3x_2 &= 1 \\ 2x_1 + hx_2 &= k \end{aligned}$

In Exercises 21 and 22, mark each statement True or False. Justify each answer.<sup>4</sup>

21. a. In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.
- b. The row reduction algorithm applies only to augmented matrices for a linear system.
- c. A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.
- d. Finding a parametric description of the solution set of a linear system is the same as *solving* the system.
- e. If one row in an echelon form of an augmented matrix is  $[0 \ 0 \ 0 \ 5 \ 0]$ , then the associated linear system is inconsistent.
22. a. The reduced echelon form of a matrix is unique.
- b. If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.
- c. The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.
- d. A general solution of a system is an explicit description of all solutions of the system.
- e. Whenever a system has free variables, the solution set contains many solutions.

23. Suppose the coefficient matrix of a linear system of four equations in four variables has a pivot in each column. Explain why the system has a unique solution.

24. Suppose a system of linear equations has a  $3 \times 5$  augmented matrix whose fifth column is not a pivot column. Is the system consistent? Why (or why not)?

<sup>4</sup> True/false questions of this type will appear in many sections. Methods for justifying your answers were described before Exercises 23 and 24 in Section 1.1.