Prof. Ming Gu, 861 Evans, tel: 2-3145
Office Hours: TuTh 12:30-2:00PM
Email: mgu@math.berkeley.edu
http://www.math.berkeley.edu/∼mgu/MA54M

Math54 Sample Midterm II, Fall 2007

This is a closed everything exam, except a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	5	
2	19	
3	19	
4	19	
5	19	
6	19	
Total	100	

1. (5 Points) Write your personal information below.

   Your Name: _____

   Your GSI: _____

   Your SID: _____

2. Consider the following map from $\mathbf{R}^2$ to $\mathbf{R}^2$:

$$\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y-1 \\ x-y+1 \end{pmatrix}.$$

Is $\mathbf{T}$ a linear transform? Explain.

3. Is the following identity true for all pairs of invertible matrices $A, B \in \mathcal{R}^{n\times n}$?

$$\left((A\,B)^{-1}\right)^T = \left(A^{-1}\right)^T \left(B^{-1}\right)^T.$$

Explain your answer.

4. Give an explicit formula for the components of the vector

$$\begin{pmatrix} -3 & 4 \\ 4 & 3 \end{pmatrix}^k \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

for any integer $k > 0$.

5. Show that the following function

$$\mathbf{x} \cdot \mathbf{y} = 3x_1y_1 + 2x_2y_2$$

is an inner product on $\mathbf{R}^2$.

6. Let

$$V = \mathbf{span}\left(\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\right).$$

Find an orthonormal basis for the orthogonal complement of V.