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Math221: Matrix Computations
Homework #3 Solutions

- **2.7:** Since A is nonsingular, all diagonal entries of D must be non-zero. Define $U = D\,M^T$, it follows from Theorem 2.4 that all leading principal submatrices of A are non-singular and the LU factorization of A uniquely exists as $A = L\,U$, with U defined as above. On the other hand, since A is symmetric, $A = A^T = M\left(DL^T\right)$ is another LU factorization for A. Because of uniqueness, we must have $L = M$.

- **2.13 (3):** Define $y_0 = c$ and

  $$y_{k+1} = y_k - A^{-1}\left(By_k - c\right), \quad k = 0, 1, 2, \cdots.$$

  Then

  $$y_{k+1} - B^{-1}c = \left(I - A^{-1}B\right)\left(y_k - B^{-1}c\right).$$

  Hence

  $$\|y_{k+1} - B^{-1}c\| \le \|A^{-1}\|\|A - B\|\|y_k - B^{-1}c\|.$$

  For $\|A - B\|$ sufficiently small, $\|A^{-1}\|\|A - B\| < 1$ and hence the limit of the sequence $\{y_k\}$ is $B^{-1}c$.

- **2.18:** We will assume that all the leading principal submatrices of A are non-singular. If this is not the case, a simple continuity argument would make up for the gap left by this assumption.

  Assume that we have performed k steps of Gaussian elimination, so that

  $$A = \begin{pmatrix} L_{11} & \\ L_{21} & I \end{pmatrix}\begin{pmatrix} U_{11} & U_{12} \\ & \hat{S} \end{pmatrix},$$

  where $\hat{S}$ is the matrix that overwrites A_{22}.

  On the other hand, direct block elimination also gives

  $$A = \begin{pmatrix} I & \\ A_{2,1}A_{11}^{-1} & I \end{pmatrix}\begin{pmatrix} A_{11} & A_{12} \\ & S \end{pmatrix}.$$

Replacing A_{11} by its LU factorization $A_{11} = L_{11}U_{11}$, and by the uniqueness of the LU factorization, we can rewrite the above equation as

$$A = \begin{pmatrix} L_{11} & \\ L_{21} & I \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} \\ & \hat{S} \end{pmatrix}.$$

Hence $\hat{S} = S$.