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Math 221, Fall 2008: Matrix Computations Term Projects

This is a tentative list of term projects for Math 221. Students are strongly encouraged to work on them as groups of up to 3 people. At the end of the semester, each group is to present their work on their project. We should start gathering and reading reference papers right away. We will have weekly meetings during office hours on project progress.

I myself do not know much about the issues involved. Some of the topics here are still under active attention of the research community.

1. **L-curve:** A method of much academic discussion in solving the ill-conditioned least squares problems is to use the **L-curve**. This project aims to summarize recent academic discussions on this subject.

References:

P. C. Hansen: *Analysis of discrete ill-posed problems by means of the L-curve*, SIAM Review, **34** (1992), pp. 561-580.

P. C. Hansen: *The L-curve and its use in the numerical treatment of inverse problems*, in *Computational Inverse Problems in Electrocardiology*, ed. P. Johnston, Advances in Computational Bioengineering, 2000.

2. **Symmetric Definite Generalized Eigenvalue Problem:** Let B be a symmetric positive definite matrix (SPD). It is known that there exists a lower triangular matrix L such that $B = LL^T$. The matrix-matrix product on the right hand side is the Cholesky factorization of B .

Let A be a symmetric matrix and B an SPD matrix. The generalized eigenvalue problem is to find pairs (λ, x) such that

$$Ax = \lambda Bx, \quad \text{where } \lambda \text{ is scalar and } x \neq 0. \quad (1)$$

With $B = LL^T$, this problem can be rewritten as

$$\hat{A}\hat{x} = \lambda\hat{x},$$

where $\hat{A} = L^{-1}AL^{-T}$ is symmetric. This is a standard symmetric eigenvalue problem for which efficient and reliable algorithms exist. The problem with this approach for solving (1) is that computing \hat{A} from L can be numerically unstable.

The aim of this project is to research the literature on existing methods for solving this problem and analyze their efficiency and stability properties.

References:

S. Chandrasekaran, *An efficient and stable algorithm for the symmetric-definite generalized eigenvalue problem*, SIAM J. Mat. Anal. Appl., Vol. 21, pp. 1202-1228, 2000.

W. Kahan, personal communications.

3. **Backward Error for Least Squares:** Given an alleged solution \hat{x} to the least squares problem $\min_x \|Ax - b\|_2$, we would like to find a perturbation ΔA with the smallest 2-norm such that \hat{x} is the exact solution to the perturbed least squares problem $\min_x \|(A + \Delta A)x - b\|_2$.

The aim of this project is to research the literature on this subject, in terms of theoretical solutions and practical algorithms.

References: B. Walden, R. Karlson, J.-G. Sun, *Optimal backward perturbation bounds for the linear least squares problem*, Numerical Linear Algebra with Applications, Vol.2, issue 3, pages 271-286, 2005.

4. **Geodetic Non-linear Least Squares Problem:** This problem is discussed in the text as an example of the least squares problem. The goal of this project is to find out the current status of this problem, in term of how it is posed and how it is solved.

References: Google??

5. **Quadratic Eigenvalue Problem:** Given square matrices M , B , and K , the quadratic eigenvalue problem is to find scalar λ and vector $x \neq 0$ such that

$$(\lambda^2 M + \lambda B + K)x = 0.$$

The goal of this project is to find out recent algorithms developed specifically for solving this problem.

References: See wikipedia.org entry for this problem.

6. **Fast Matrix-Matrix Multiplication:** Strassen's surprising algorithm takes about $O(n^{2.7})$ flops to multiple two $n \times n$ matrices. Yet it has long ceased to be the world's fastest matrix-matrix multiplication algorithm. In this project, we would like to find out the current fastest algorithms and how they work.

References:

B. Kakaradov, *Ultra-Fast Matrix Multiplication*: surj.stanford.edu/2004/pdfs/kakaradov.pdf