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Math221: Matrix Computations
Homework #10, Due Nov. 15, 2007

- Generate symmetric tridiagonal matrices of various dimensions, and run matlab codes `SQR.m` and `rayleigh.m` (available on class website). Demonstrate local cubic convergence on your matrices. Do Problem 5.13.
- Let $A = Q\Lambda Q^*$ be the eigendecomposition of A, with $Q = [q_1, \cdots, q_n]$, and let the initial vector $x_0 = q_1 + q_2$. Show that RQI fails to converge in exact arithmetic. Run `rayleigh.m` with this initial vector to see what it does in finite precision.
- Let $B \in \mathbf{R}^{n\times n}$ be an upper bidiagonal matrix. Find explicit formulas for its inverse.
- Generate upper bidiagonal matrices of various dimensions, and run matlab code `BiSVD.m` (available on class website) to compute their smallest singular values. You should try different scalings on the diagonal entries so the smallest singular values can be really tiny ($10^{-100} - 10^{-50}$, for example).

  To check that these are indeed very accurate singular values, we use the formula

$$1/\sigma_{\max}\left(B^{-1}\right) = \sigma_{\min}\left(B\right). \tag{1}$$

  The matlab `svd` function is backward stable. We generate B^{-1} explicitly using the explicit formulas. This way the largest singular value of B^{-1} is computed to full machine precision. Compare $1/\sigma_{\max}\left(B^{-1}\right)$ with the singular values computed using `BiSVD.m` to show that `BiSVD.m` is highly accurate even for tiny singular values.
- Problems 5.27, 5.28.