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Math221: Matrix Computations
Homework #6, Due Oct. 11, 2007

- Problems 3.8, 3.12, 3.15, 3.16, 3.18.
- For any non-zero vector $x = (x_1, \cdots, x_n)^T$, the standard way to compute the Householder transformation is to compute $\widetilde{u} = (x_1 - c, x_2, \cdots, x_n)^T$ with $c = -\mathbf{sign}(x_1)\|x\|_2$ and $u = \widetilde{u}/\|\widetilde{u}\|_2$ so that
$$\left(I - 2uu^T\right) x = (c, 0, \cdots, 0)^T.$$
The special sign of c ensures that $\widetilde{u}$ and u are computed to full relative accuracy.

  However, the sign choice in c is actually not necessary. Let $c = \|x\|_2$. Show that $\widetilde{u}$, and hence u, can still be computed to full relative accuracy with a computationally different but mathematically equivalent formula. Perform an error analysis to support your claim. You can assume the square root function is always accurate to full relative accuracy. Write a matlab code to demonstrate that the straightforward formula for computing $\widetilde{u}$ can be unstable and yours is always stable. The matlab code housetest.m on the class website generates vectors that fail the straightforward formula.
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  - Let $c^2 + s^2 = 1$ and let $q \in \mathbf{R}^{n-1}$ be a unit vector. Find vectors $r, u, v \in \mathbf{R}^{n-1}$ so that the matrix
$$Q = \begin{pmatrix} c & r^T \\ sq & I - uv^T \end{pmatrix} \in \mathbf{R}^{n \times n}$$
is an orthogonal matrix.
  - For any non-zero vector x, find a Q matrix of the form above such that $Qx = (\|x\|_2, 0, \cdots, 0)^T$.
  - Develop a QR factorization algorithm that is based on the Q matrices, and show that it is stable. Compare the cost of your algorithm with that based on Householder transformations.
  - Correctly implement your algorithm in matlab.