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Math221: Matrix Computations

Homework #3 Solutions

- **2.13 (3):** Define $y_0 = c$ and

$$y_{k+1} = y_k - A^{-1}\left(B y_k - c\right), \quad k = 0, 1, 2, \cdots.$$

Then

$$y_{k+1} - B^{-1}c = \left(I - A^{-1}B\right)\left(y_k - B^{-1}c\right).$$

Hence

$$\|y_{k+1} - B^{-1}c\| \le \|A^{-1}\|\|A - B\|\|y_k - B^{-1}c\|.$$

For $\|A - B\|$ sufficiently small, $\|A^{-1}\|\|A - B\| < 1$ and hence the limit of the sequence $\{y_k\}$ is $B^{-1}c$.

- **2.18:** We will assume that all the leading principal submatrices of A are non-singular. If this is not the case, a simple continuity argument would make up for the gap left by this assumption.

Assume that we have performed k steps of Gaussian elimination, so that

$$A = \begin{pmatrix} L_{11} & \\ L_{21} & I \end{pmatrix}\begin{pmatrix} U_{11} & U_{12} \\ & \hat{S} \end{pmatrix},$$

where $\hat{S}$ is the matrix that overwrites A_{22}.

On the other hand, direct block elimination also gives

$$A = \begin{pmatrix} I & \\ A_{2,1}A_{11}^{-1} & I \end{pmatrix}\begin{pmatrix} A_{11} & A_{12} \\ & S \end{pmatrix}.$$

Replacing A_{11} by its LU factorization $A_{11} = L_{11}U_{11}$, and by the uniqueness of the LU factorization, we can rewrite the above equation as

$$A = \begin{pmatrix} L_{11} & \\ L_{21} & I \end{pmatrix}\begin{pmatrix} U_{11} & U_{12} \\ & \hat{S} \end{pmatrix}.$$

Hence $\hat{S} = S$.

- Problems **2.20:**
  - **(a):** Compute GEPP $A = PLU$. Solving $A^k x = b$ then involves k permutations as well as k forward and backward substitutions. Total cost: $2/3n^3 + kn^2 + O(n^2)$.
  - **(b):** Compute GEPP $A = PLU$, and solve for $A^{-1}b$.
  - **(c):** Compute GEPP $A = PLU$. Solving $AX = B$ then involves m permutations as well as m forward and backward substitutions. Total cost: $2/3n^3 + mn^2 + O(n^2)$.