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Math221: Matrix Computations Selected Solutions to Homework #1, 2007

• Problem 1.6:

$$\begin{split} \left\| E\left(I - \frac{ss^{T}}{s^{T}s}\right) \right\|_{F}^{2} &= \operatorname{tr}\left(\left(E\left(I - \frac{ss^{T}}{s^{T}s}\right) \right) \left(E\left(I - \frac{ss^{T}}{s^{T}s}\right) \right)^{T} \right) \\ &= \operatorname{tr}\left(\left(E\left(I - \frac{ss^{T}}{s^{T}s}\right) \right) E^{T} \right) \\ &= \operatorname{tr}\left(EE^{T} \right) - \operatorname{tr}\left(\frac{Es(Es)^{T}}{s^{T}s} \right) = \|E\|_{F}^{2} - \left(\frac{\|Es\|_{2}^{2}}{s^{T}s} \right) \end{split}$$

- Problem 1.9: We will only consider the computations with tiny x > 0. Let ϵ be the machine precision and let $x = (k + f)\epsilon$, where k is an integer satisfying $10 > k \ge 0$ and f is the fraction with $1 > f \ge 0$. In all cases, we need to compute d = 1 + x. We assume that $\mathbf{fl}(d)$ is obtained via rounding to the nearest. This implies that $\mathbf{fl}(d) = 1 + k\epsilon$ if f < 1/2 and $\mathbf{fl}(d) = 1 + (k + 1)\epsilon$ otherwise. This discontinuity in floating point computation of 1 + x is the cause of the large errors in left plots. On the right plots, since d = 1 + x has already been computed and is stored as $\mathbf{fl}(d)$, both $\log(\mathbf{fl}(d))$ and $\mathbf{fl}(d) 1$ can be computed highly accurately with respect to $\mathbf{fl}(d)$.
- overflow/underflow protection in quadroot.m: On the class website, there is a modified version of quadroot.m as well as a paper on solving for quadratic roots by George Forsythe.