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Math 221: Matrix Computations Term Projects

This is a tentative list of term projects for Math 221. I will probably modify them or add more projects as the course progresses. Students are strongly encouraged to work on them as groups of up to 4 people. At the end of the semester, each group is to present their work on their project. One should start working on their project as soon as possible. We should start gathering and reading reference papers right away. After the Midterm, we should have weekly meetings during office hours on project progress.

I myself do not have full knowledge of all the issues involved. Some of the topics here are still under active attention of the research community.

1. **Orbit of Ceres:** Gauss invented the least squares (LS) method to recover the orbit of dwarf planet Ceres. The aim of this project is to recover exactly what Gauss's method was and what his LS problem was. Write a code to redo what Gauss did by hand 200 years ago.

   References:

   J. Tennenbaum and B. Director, *How Gauss determined the orbit of Ceres,* at `www.schillerinstitute.org/fid_97-01/982_orbit_ceres.pdf`.

   People in this project:
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2. **Symmetric Definite Generalized Eigenvalue Problem:** Let B be a symmetric positive definite matrix (SPD). It is known that there exists a lower triangular matrix L such that $B = LL^T$. The matrix-matrix product on the right hand side is called the Cholesky factorization of B.

   Let A be a symmetric matrix and B an SDP matrix. The generalized eigenvalue problem is to find pairs (λ, x) such that

   $$Ax = \lambda Bx, \quad \text{where } \lambda \text{ is scalar and } x \neq 0. \tag{1}$$

With $B = LL^T$, this problem can be rewritten as

$$\widehat{A}\widehat{x} = \lambda\widehat{x},$$

where $\widehat{A} = L^{-1}AL^{-T}$ is symmetric. This is a standard symmetric eigenvalue problem for which efficient and reliable algorithms exist. The problem with this approach for solving (1) is that computing $\widehat{A}$ from L can be numerically unstable.

The aim of this project is to research the literature on existing methods for solving this problem and analyze their efficiency and stability properties.

References:

S. Chandrasekaran, *An efficient and stable algorithm for the symmetric-definite generalized eigenvalue problem*, SIAM J. Mat. Anal. Appl., Vol. 21, pp. 1202-1228, 2000.

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3. **Error Estimation and Condition Estimation:** Given any alleged solution $\widehat{x}$ to the linear equations $Ax = b$, define the residual as $r = A\widehat{x} - b$. Let $\delta x = \widehat{x} - x$ be the difference between $\widehat{x}$ and the true solution. Then

$$\delta x = A^{-1}r. \tag{2}$$

This simple equation seems to suggest an easy way to bound $\|\delta x\|$ given a factorization of A. But the truth is much messier. The problems are

(a) The factorization might not be backward stable.

(b) There might be rounding errors in computing r.

(c) There might be rounding errors in solving for δx from the given factorization and computed r.

Because of these problems, bounding $\|\delta x\|$ correctly is much harder than solving for δx via equation (2).

Modify the LAPACK code `dgesvx` to compute error bounds based on (2) and compare them with those produced by `dgesvx`. Analyze the reliability of your bounds. We will assume LU with partial pivoting is used for the factorization.

References: See Demmel's Book.

J. Demmel, Y. Hida, W. Kahan, X. S. Li, S. Mukherjee, E. J. Riedy, *Error bounds from extra precise iterative refinement*, `www.eecs.berkeley.edu/Pubs/TechRpts/2004/CSD-04-1344.pdf`.

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